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1 Introduction

This document shows examples of use of Autodesk.CodeChecking.Concrete named later “Component”.

Each use case includes paragraphs:

- Subject – short description of what we want to calculate and what data do we need
- Summary – detailed list of available results obtained for the list of data
- Data – example data
- Search for – searched results
- Results from Component – results obtained from Component
- Results (manual calculations) – results obtained using manual calculations with calculation path. Some examples (Case 14, Cases 16-18) compare Component results with results from the literature (design manuals) and other program.

Examples below have physical sense for metric system units.

To see calculation methods, assumptions used by this Component and rules concerning units, please refer to the User Manual of the Code Checking Framework SDK.

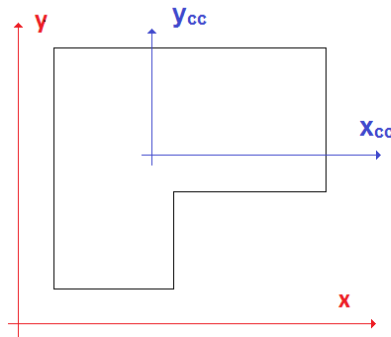
Cases shown in this document are related to the C# example provided with this SDK.

Every case has its own implementation in the single *.cs file with the name corresponding to the example (e.g. file “Case01.cs” corresponding to example Case 1). The project includes a dialog which allows selection of examples, run one or all of the examples and presents the results of calculation.

Main symbols used in this document:

x – the first axis of the initial coordinate system (x, y)

y – the second axis of the initial coordinate system (x, y)



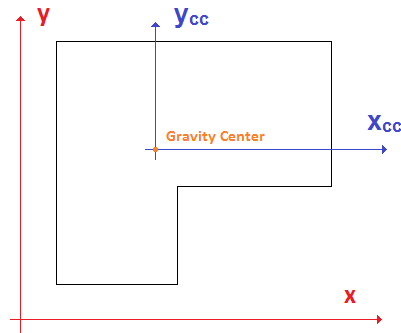
A_c – concrete gross section area or area of contour under compressive stress

x_{cc} – concrete gross section gravity center first coordinate in (x, y) coordinate system

y_{cc} – concrete gross section gravity center second coordinate in (x, y) coordinate system

x_{cc} - central axis of concrete gross section parallel to x axis or the first coordinate in (x_{cc}, y_{cc}) coordinate system

y_{cc} - central axis of concrete gross section parallel to y axis or the second coordinate in (x_{cc}, y_{cc}) coordinate system



I_{cx} – moment of inertia of concrete gross section with respect to x_{cc} axis

I_{cy} – moment of inertia of concrete gross section with respect to y_{cc} axis

A_s – reinforcement bars area

x_s – reinforcement bars gravity center first coordinate in (x,y) coordinate system

y_s – reinforcement bars gravity center second coordinate in (x,y) coordinate system

I_{sx} – moment of inertia of reinforcement bars with respect to x_{cc} axis

I_{sy} – moment of inertia of reinforcement bars with respect to y_{cc} axis

A_{eff} – reduced cross section area

x_c – reduced cross section gravity center first coordinate in (x,y) coordinate system

y_c – reduced cross section gravity center second coordinate in (x,y) coordinate system

I_{effx} – moment of inertia of reduced cross section with respect to x axis

I_{effy} – moment of inertia of reduced cross section with respect to y axis

N – internal axial force in reduced cross section

M_x – moment of internal forces in reduced section with respect to x_{cc} axis

M_y – moment of internal forces in reduced section with respect to y_{cc} axis

N_s – internal axial force in reinforcing bars

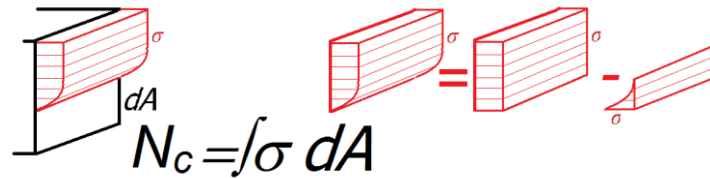
M_{xs} – moment of internal forces in reinforcing bars with respect to x_{cc} axis

M_{ys} – moment of internal forces in reinforcing bars with respect to y_{cc} axis

x_{cg} – center of gravity of stresses in the concrete with respect to x_{cc} axis

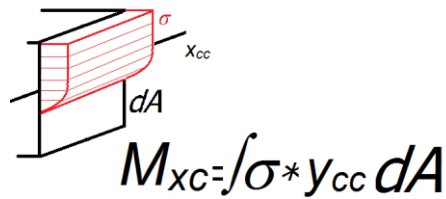
y_{cg} – center of gravity of stresses in the concrete with respect to y_{cc} axis

N_c – internal axial force in the concrete part of a section, calculated as a volume of solid of stresses as shown below:



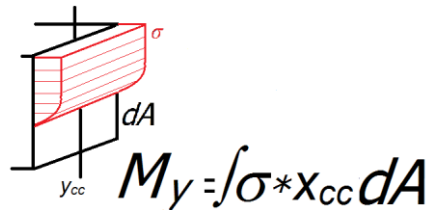
$$N_c = \int \sigma dA$$

M_{xc} – moment of internal forces in the concrete part of a section with respect to x_{cc} axis



$$M_{xc} = \int \sigma * y_{cc} dA$$

M_{yc} – moment of internal forces in the concrete part of a section with respect to y_{cc} axis



$$M_y = \int \sigma * x_{cc} dA$$

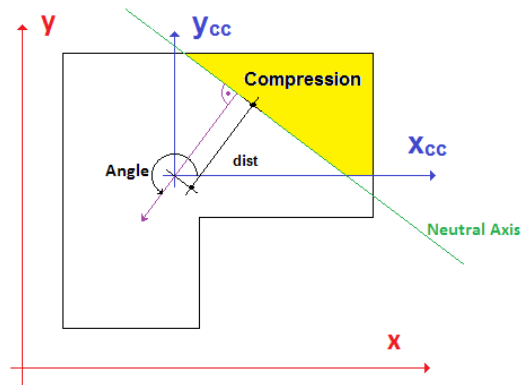
Eps_bot – a strain in the most tensioned point of concrete section

Eps_top – a strain in the most compressed point of concrete section

Eps_sbot – a strain in the most tensioned rebar

Eps_stop – a strain in the most compressed rebar

Angle – angle between horizontal axis and normal to neutral axis directed to the most tensioned fiber



dist – distance between neutral axis and a gravity center of concrete gross section

N – internal axial force in cross section at the failure

M_{xf} – moment of internal forces in section with respect to x_{cc} axis at the failure

M_{yf} – moment of internal forces in section with respect to y_{cc} axis at the failure

sigma, sigma_i, Sigma, Sigma_{max}, Sigma_{min} – stress in rebar or in concrete, description in the text

eps, eps_i – stress in rebar or in concrete, description in the text

2 Case 1: Cross section characteristics

2.1 Subject:

Read cross section geometry, positions of reinforcing bars inside the section, parameters of concrete and steel and give cross section characteristics

2.2 Summary:

This sample gives as results the cross section characteristics.

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel.

Based on input data, Component gives as output results:

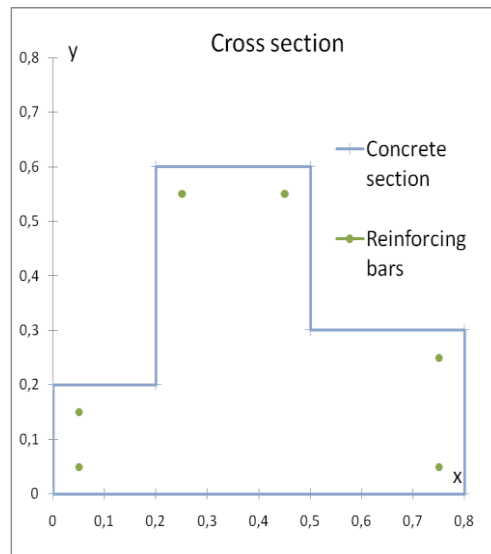
- cross section characteristics:
 - Concrete section gross area (A_c) with the coordinates of its gravity center (x_{cc} , y_{cc})
 - Moments of inertia (I_{cx} , I_{cy})
 - Reinforcing bars area (A_s) with coordinates of its gravity center (x_s , y_s)
 - Moments of inertia (I_{sx} , I_{sy})
 - Reduced cross section characteristics (A_{eff} , x_c , y_c , I_{effx} , I_{effy}).

2.3 Data:

- Geometry:
 - Concrete section:

$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.2$
$x = 0.2$	$y = 0.2$
$x = 0.2$	$y = 0.6$
$x = 0.5$	$y = 0.6$
$x = 0.5$	$y = 0.3$
$x = 0.8$	$y = 0.3$
$x = 0.8$	$y = 0.0$
 - Reinforcing bars:

$x = 0.05$	$y = 0.05$	$\phi = 0.010$
$x = 0.75$	$y = 0.05$	$\phi = 0.012$
$x = 0.75$	$y = 0.25$	$\phi = 0.020$
$x = 0.45$	$y = 0.55$	$\phi = 0.050$
$x = 0.25$	$y = 0.55$	$\phi = 0.020$
$x = 0.05$	$y = 0.15$	$\phi = 0.015$



- Concrete parameters:
Modulus of elasticity $E_c = 30e9$
- Steel parameters:
Modulus of elasticity $E_s = 200e9$

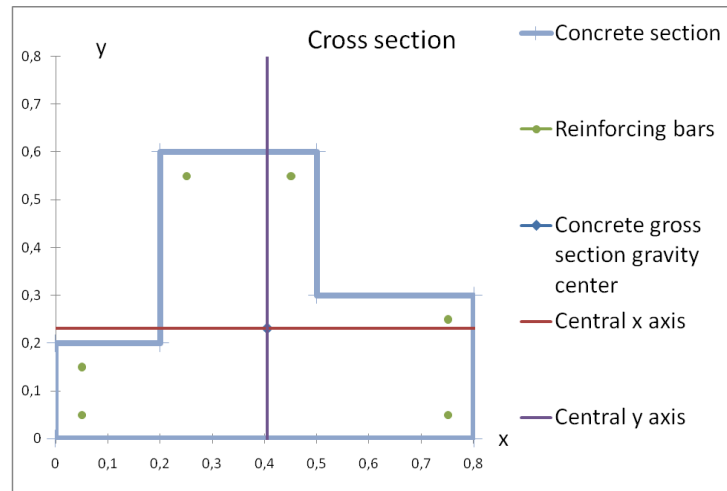
2.4 Search for:

- Concrete gross section characteristics: A_c , x_{Cc} , y_{Cc} , I_{cx} , I_{cy} ,
- Reinforcing bars characteristics: A_s , x_s , y_s , I_{sx} , I_{sy} ,
- Reduced cross section characteristics: A_{eff} , x_c , y_c , I_{effx} , I_{effy} .

2.5 Results from Component

- Concrete gross section characteristics:
 $A_c = 0.310000$
 $x_{Cc} = 0.40483871$
 $y_{Cc} = 0.23064516$
 $I_{cx} = 0.008342$
 $I_{cy} = 0.011826$
- Reinforcing bars characteristics:
 $A_s = 0.002960$
 $x_s = 0.43758291$
 $y_s = 0.46191298$
 $I_{sx} = 0.000240$
 $I_{sy} = 0.000095$
- Reduced cross section characteristics:
 $A_{eff} = 0.326774$
 $x_c = 0.40651956$
 $y_c = 0.24251681$
 $I_{effx} = 0.009701$

$$I_{\text{effy}} = 0.012362$$



2.6 Results (manual calculations):

- Concrete gross section characteristics:

Area:

$$A_c = 0.2 \cdot 0.2 + 0.3 \cdot 0.6 + 0.3 \cdot 0.3 = \underline{0.31}$$

Static moments:

$$S_{cx} = 0.2 \cdot 0.2 \cdot 0.1 + 0.3 \cdot 0.6 \cdot 0.3 + 0.3 \cdot 0.3 \cdot 0.15 = \underline{0.0715}$$

$$S_{cy} = 0.2 \cdot 0.2 \cdot 0.1 + 0.3 \cdot 0.6 \cdot 0.35 + 0.3 \cdot 0.3 \cdot 0.65 = \underline{0.1255}$$

Gravity center:

$$x_{Cc} = S_{cy} / A_c = \underline{0.40483871}$$

$$y_{Cc} = S_{cx} / A_c = \underline{0.23064516}$$

Moments of inertia:

$$I_x = 0.2 \cdot 0.2^3 / 12 + 0.2 \cdot 0.2 \cdot 0.1^2 + 0.3 \cdot 0.6^3 / 12 + 0.3 \cdot 0.6 \cdot 0.3^2 + 0.3 \cdot 0.3^3 / 12 + 0.3 \cdot 0.3 \cdot 0.15^2 = \underline{0.024833}$$

$$I_y = 0.2 \cdot 0.2^3 / 12 + 0.2 \cdot 0.2 \cdot 0.1^2 + 0.6 \cdot 0.3^3 / 12 + 0.3 \cdot 0.6 \cdot 0.35^2 + 0.3 \cdot 0.3^3 / 12 + 0.3 \cdot 0.3 \cdot 0.65^2 = \underline{0.062633}$$

Central moments of inertia:

$$I_{cx} = I_x - A_c \cdot y_{Cc}^2 = \underline{0.008342}$$

$$I_{cy} = I_y - A_c \cdot x_{Cc}^2 = \underline{0.011826}$$

- Reinforcing bars characteristics:

i	x	y	ϕ	$A_s = \pi \cdot \phi^2 / 4$	Static moment $S_{sy} = A_s \cdot x$	Static moment $S_{sx} = A_s \cdot y$	Coordinates in x_{cc}, y_{cc} system $x_{cc} = x - x_{Cc} \quad y_{cc} = y - y_{Cc}$		$I_{sx} = \pi \cdot \phi^4 / 64 + A_s \cdot y_{cc}^2$	$I_{sy} = \pi \cdot \phi^4 / 64 + A_s \cdot x_{cc}^2$
1	0,05	0,05	0,01	0,000079	0,000004	0,000004	-0,3548	-0,1806	0,000003	0,000010
2	0,75	0,05	0,012	0,000113	0,000085	0,000006	0,3452	-0,1806	0,000004	0,000013
3	0,75	0,25	0,02	0,000314	0,000236	0,000079	0,3452	0,0194	0,000000	0,000037
4	0,45	0,55	0,05	0,001963	0,000884	0,001080	0,0452	0,3194	0,000201	0,000004
5	0,25	0,55	0,02	0,000314	0,000079	0,000173	-0,1548	0,3194	0,000032	0,000008
6	0,05	0,15	0,015	0,000177	0,000009	0,000027	-0,3548	-0,0806	0,000001	0,000022
Total:				0,002960	0,001295	0,001367			0,000240	0,000095

Area:

$$A_s = 0.002960$$

Static moments:

$$S_{sx} = 0.001367$$

$$S_{sy} = 0.001295$$

Gravity center:

$$x_s = S_{sy} / A = 0.43758291$$

$$y_s = S_{sx} / A = 0.46191298$$

Moments of inertia:

$$I_{sx} = 0.000240$$

$$I_{sy} = 0.000095$$

- Reduced cross section characteristics:

Area:

$$A_{eff} = A_c + A_s \cdot (E_s / E_c - 1) = 0.326774$$

Static moments:

$$S_x = S_{cx} + S_{sx} \cdot (E_s / E_c - 1) = 0.079246$$

$$S_y = S_{cy} + S_{sy} \cdot (E_s / E_c - 1) = 0.132838$$

Gravity center:

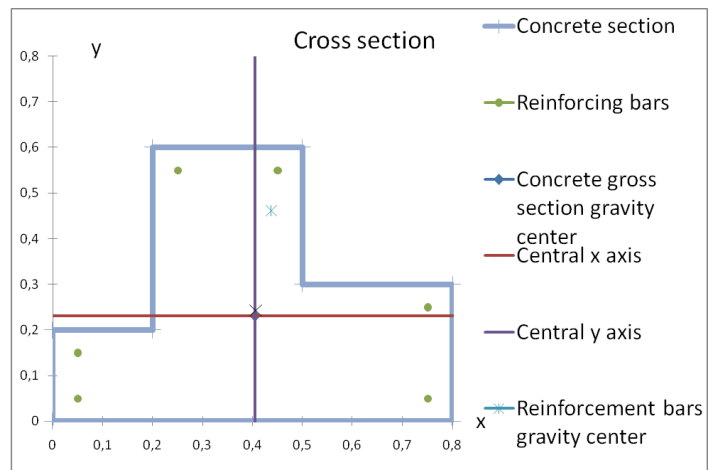
$$x_c = S_y / A = 0.4065$$

$$y_c = S_x / A = 0.2425$$

Moments of inertia:

$$I_{effx} = I_{cx} + I_{sx} \cdot (E_s / E_c - 1) = 0.009701$$

$$I_{effy} = I_{cy} + I_{sy} \cdot (E_s / E_c - 1) = 0.012362$$



3 Case 2: Calculation of internal forces for given state of strain in the section and rectangular model of concrete

3.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, extreme strains in the section and give internal forces.

3.2 Summary:

This sample gives as results the internal forces for:

- A rectangular concrete section with symmetric reinforcement (Case 2a and Case 2b)
- A rectangular concrete shape section with asymmetric reinforcement (Case 2c)
- A rectangular model of concrete and horizontal branch model of steel
- A section under unidirectional bending with entire section under compression (Case 2a)
- A section under unidirectional bending with a part of section under compression (Case 2b and Case 2c)
- A state of strain with ultimate compressive strain in concrete (Case 2a and Case 2b)
- A strain below the ultimate compressive strain in concrete (Case 2c).

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel.

Based on input data, Component gives as output results:

- Results for reinforcement bars:
 - Axial force (N_s)
 - Moment with respect to x_{cc} axis (M_{xs})
 - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
 - Area of contour under compressive stress (A_c)
 - Center of gravity of stresses in the concrete (x_{cg} , y_{cg})
 - Axial force (N_c)
 - Moment with respect to x_{cc} axis (M_{xc})
 - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
 - Axial force (N)
 - Moment with respect to x_{cc} axis (M_x)
 - Moment with respect to y_{cc} axis (M_y)

For these results, x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

3.3 Case 2a

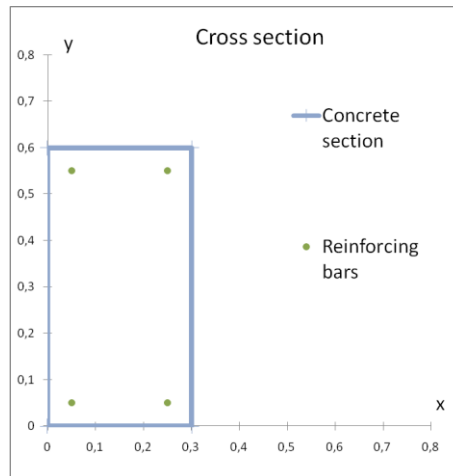
3.3.1 Data:

- Geometry:
 - Concrete section:

$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.6$
$x = 0.3$	$y = 0.6$
$x = 0.3$	$y = 0.0$

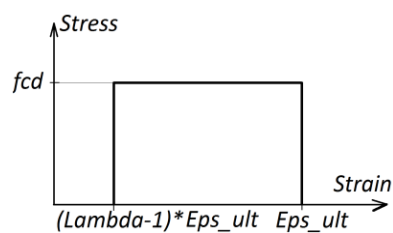
- Reinforcing bars:

$x = 0.05$	$y = 0.05$	$\phi = 0.012$
$x = 0.05$	$y = 0.55$	$\phi = 0.012$
$x = 0.25$	$y = 0.55$	$\phi = 0.012$
$x = 0.25$	$y = 0.05$	$\phi = 0.012$



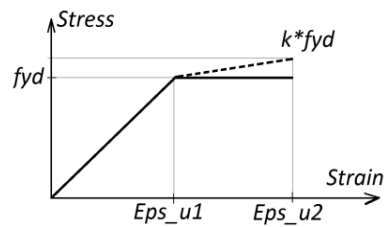
- Concrete parameters:

- Design strength $f_{cd} = 20e6$
- Effective height reduction factor $\Lambda = 0.8$
- Modulus of elasticity $E_c = 30e9$
- Strain-stress model: rectangular
- Strain ultimate limit $Eps_ult = 0.0035$

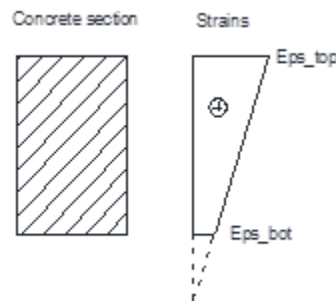


- Steel parameters:

- Design strength $f_{yd} = 500e6$
- Hardening factor $k = 1.0$
- Modulus of elasticity $E_s = 200e9$
- Strain ultimate limit $Eps_u2 = 0.075$



- Strains:
 - Bottom strain $Eps_bot = 0.0005$
 - Top strain $Eps_top = 0.0035$
 - Neutral axis angle $Angle = \pi \cdot 3 / 2$



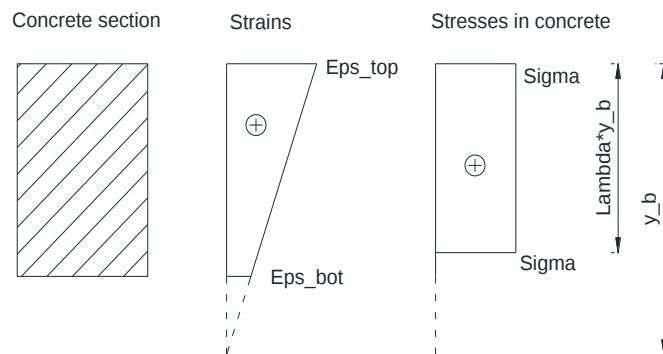
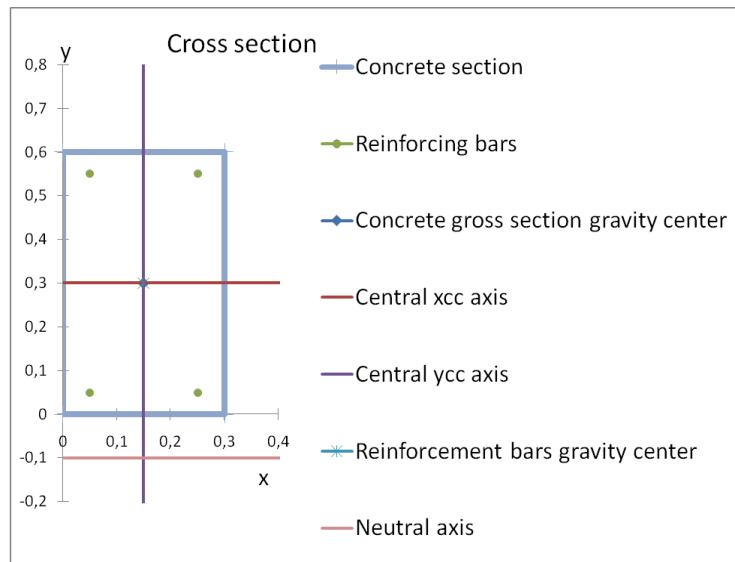
3.3.2 Search for :

- Results for reinforcement bars: N_s, M_{xs}, M_{ys}
- Results for concrete section: $A_c, (x_{cg}, y_{cg}), N_c, M_{xc}, M_{yc}$
- Results for reduced section: N, M_x, M_y

3.3.3 Results from Component

- Results for reinforcement bars:
 - $N_s = 147.03e3$
 - $M_{xs} = -19.79e3$
 - $M_{ys} = 0.00e3$
- Results for concrete section:
 - $A_c = 0.1680$
 - $(x_{cg}, y_{cg}) = (0.00000000, 0.02005400)$
 - $N_c = 3350.95e3$
 - $M_{xc} = -67.20e3$
 - $M_{yc} = 0.00e3$
- Results for reduced section:
 - $N = 3497.98e3$
 - $M_x = -86.99e3$
 - $M_y = 0.00e3$

3.3.4 Results (manual calculations):



- Results for reinforcement bars:
Distance between maximum compression corner of the section and neutral axis:
height of a section: $h = 0.6$
 $y_b = -Eps_top \cdot h / (Eps_bot - Eps_top) = 0.7$
 $Eps_u1 = f_{yd} / E_s$

Concrete gross section gravity center:

$$x_{Cc} = 0.15$$

$$y_{Cc} = 0.30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar i:

If $|eps_i| \leq f_{yd} / E_s$ then

$$sigma_i = eps_i / Eps_u1 \cdot f_{yd}$$

else

$$sigma_i = f_{yd} + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) \cdot (k - 1) \cdot f_{yd}$$

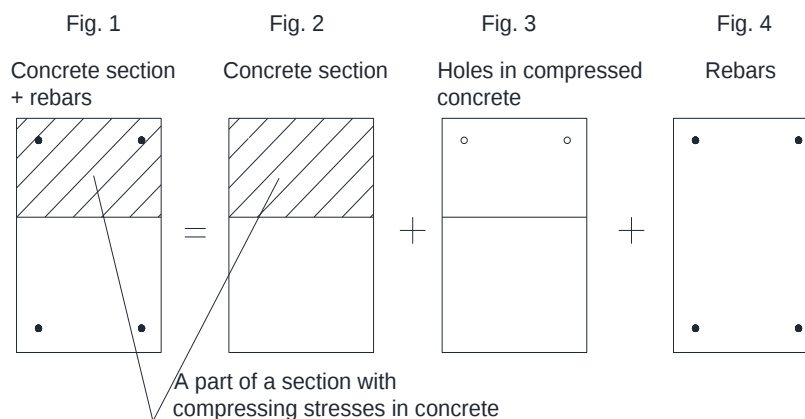
i	x	y	ϕ	Distance d from neutral axis	Strain $\epsilon_{ps} = d / y_b \cdot \epsilon_{ps_top}$	Stresses $\sigma / 10e6$	Force in bar $N = \sigma \cdot \pi \cdot \phi^2 / 4$	Positions in central axes coordinate system x_{cc} y_{cc}		$M_{xs} = -N \cdot y_{cc}$	$M_{ys} = N \cdot x_{cc}$
1	0,05	0,05	0,012	0,150	0,00075	150	16965	-0,1	-0,25	4241	-1696
2	0,05	0,55	0,012	0,650	0,00325	500	56549	-0,1	0,25	-14137	-5655
3	0,25	0,55	0,012	0,650	0,00325	500	56549	0,1	0,25	-14137	5655
4	0,25	0,05	0,012	0,150	0,00075	150	16965	0,1	-0,25	4241	1696
						Sum of forces in bars:	147027			-19792	0

$$N_s = 147.03e3$$

$$M_{xs} = -19.79e3$$

$$M_{ys} = 0.00e3$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing below).



Reduction of forces carried by the concrete section is calculated as follows:

i	x	y	ϕ	Positions in central axes coordinate system x_{cc} y_{cc}		Area of concrete with compression stress cut from the section in place of bars	Force N_{sc} in concrete in place of bars $N_{sc} = f_{cd} \cdot \epsilon_{ps_top} / \epsilon_{ps_ult} \cdot \text{Area}$	$M_{xsc} = -N_{sc} \cdot y_{cc}$	$M_{ysc} = N_{sc} \cdot x_{cc}$
1	0,05	0,05	0,012	-0,1	-0,25	0,000113097	2262	565	-226
2	0,05	0,55	0,012	-0,1	0,25	0,000113097	2262	-565	-226
3	0,25	0,55	0,012	0,1	0,25	0,000113097	2262	-565	226
4	0,25	0,05	0,012	0,1	-0,25	0,000113097	2262	565	226
						Sum of forces:	9048	0	0

Reduction forces in concrete:

$$N_{sc} = 9.05e3$$

$$M_{xsc} = 0.00e3$$

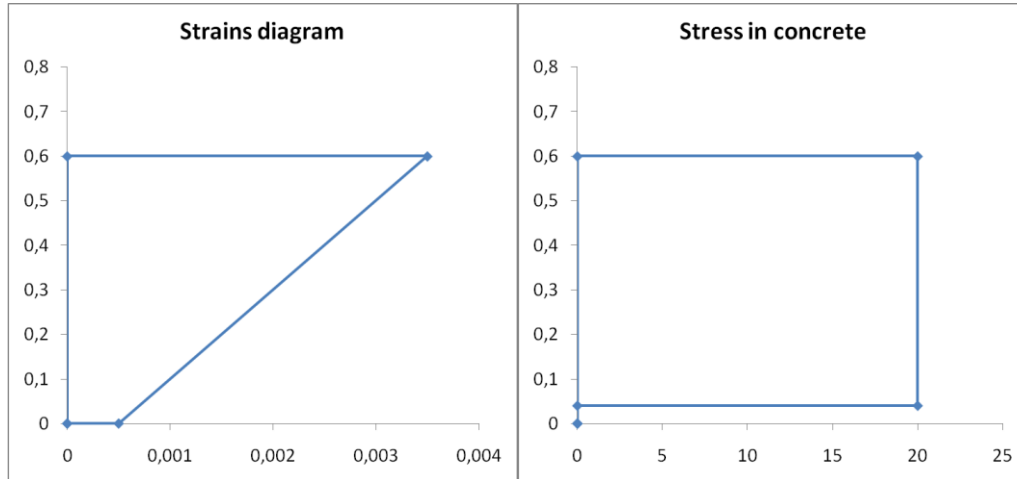
$$M_{ysc} = 0.00e3$$

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see fig. 3 on drawing above) in central axes coordinate system:

$$x_{scg} = \sum(x_{cc_i} \cdot Area_i) / \sum(Area_i) = 0.00$$

$$y_{scg} = \sum(y_{cc_i} \cdot Area_i) / \sum(Area_i) = 0.00$$

- Results for concrete section:



Height of compressed part of the section:

$$\text{Lambda} \cdot y_b = 0.56$$

Area of contour under compressive stress:

$$A_c = 0.3 \cdot 0.56 = \underline{0.168}$$

Stress in concrete:

$$\text{Sigma} = \text{Eps_top} / \text{Eps_ult} \cdot f_{cd} = 20e6$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = ((0.3 / 2 - x_{Cc}) \cdot A_c \cdot \text{Sigma} - x_{scg} \cdot N_{sc}) / (A_c \cdot \text{Sigma} + N_{sc}) = \underline{0}$$

$$y_{cg} = ((0.6 - 0.56 / 2 - y_{Cc}) \cdot A_c \cdot \text{Sigma} - y_{scg} \cdot N_{sc}) / (A_c \cdot \text{Sigma} + N_{sc}) = \underline{0.020054}$$

Internal forces in concrete:

$$N_c = A_c \cdot \text{Sigma} - N_{sc} = \underline{3350.95e3}$$

$$M_{xc} = - N_c \cdot y_{cg} = \underline{-67.20e3}$$

$$M_{yc} = N_c \cdot x_{cg} = \underline{0.00e3}$$

- Results for reduced section:

$$N = N_c + N_s = \underline{3497.98e3}$$

$$M_x = M_{xc} + M_{xs} = \underline{-86.99e3}$$

$$M_y = M_{yc} + M_{ys} = \underline{0.00e3}$$

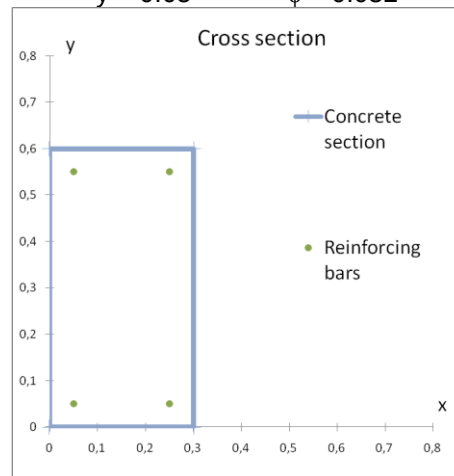
3.4 Case 2b

3.4.1 Data:

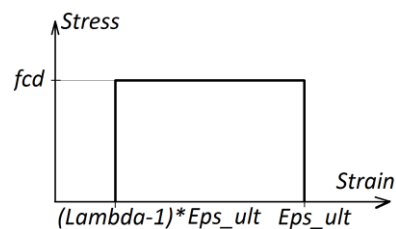
- Geometry:
 - Concrete section:

$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.6$
$x = 0.3$	$y = 0.6$
$x = 0.3$	$y = 0.0$
 - Reinforcing bars:

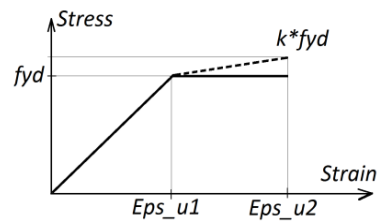
$x = 0.05$	$y = 0.05$	$\phi = 0.032$
$x = 0.05$	$y = 0.55$	$\phi = 0.032$
$x = 0.25$	$y = 0.55$	$\phi = 0.032$
$x = 0.25$	$y = 0.05$	$\phi = 0.032$



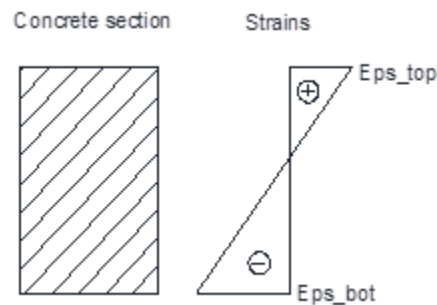
- Concrete parameters:
 - Design strength $f_{cd} = 20e6$
 - Effective height reduction factor $\Lambda = 0.8$
 - Modulus of elasticity $E_c = 30e9$
 - Strain-stress model: rectangular
 - Strain ultimate limit $Eps_ult = 0.0035$



- Steel parameters:
 - Design strength $f_{yd} = 500e6$
 - Hardening factor $k = 1.0$
 - Modulus of elasticity $E_s = 200e9$
 - Strain ultimate limit $Eps_u2 = 0.075$



- Strains:
 - Bottom strain Eps_bot = - 0.022029
 - Top strain Eps_top = 0.0035
 - Neutral axis angle Angle = $\pi \cdot 3 / 2$



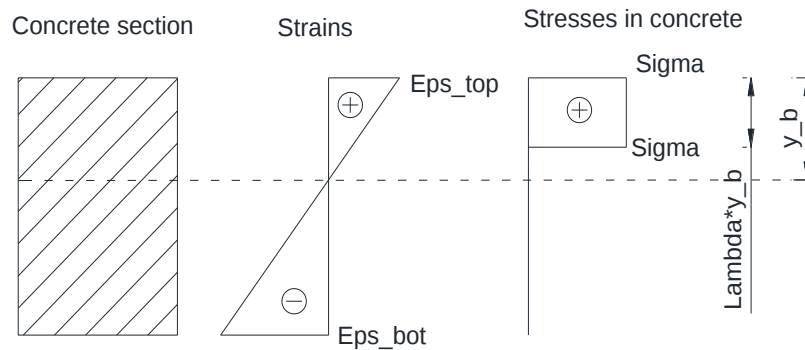
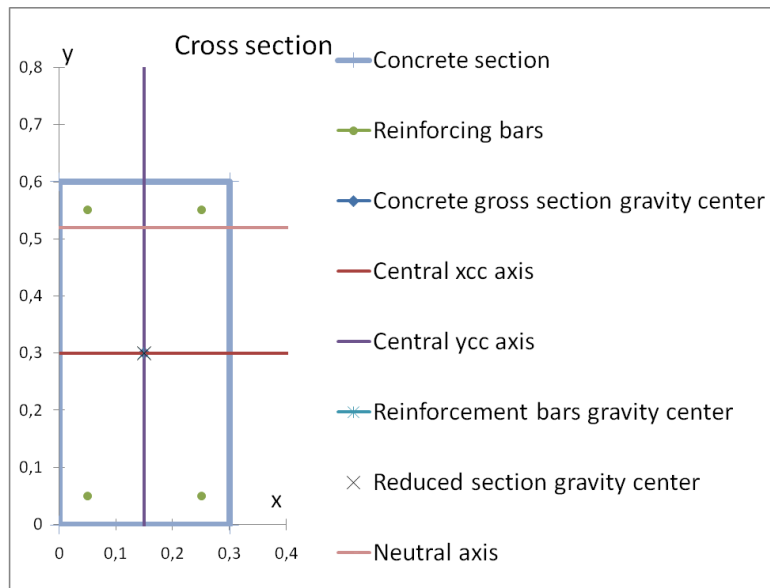
3.4.2 Search for:

- Results for reinforcement bars: N_s , M_{xs} , M_{ys}
- Results for concrete section: A_c , x_{cg} , y_{cg} , N_c , M_{xc} , M_{yc}
- Results for reduced section: N , M_x , M_y

3.4.3 Results from Component

- Results for reinforcement bars:
 - $N_s = -362.69e3$
 - $M_{xs} = -311.45e3$
 - $M_{ys} = 0.00e3$
- Results for concrete section:
 - $A_c = 0.01974$
 - $x_{cg} = 0.00000000$
 - $y_{cg} = 0.26861271$
 - $N_c = 362.68e3$
 - $M_{xc} = -97.42e3$
 - $M_{yc} = 0.00e3$
- Results for reduced section:
 - $N = -0.01e3$
 - $M_x = -408.87e3$
 - $M_y = 0.00e3$

3.4.4 Results (manual calculations):



- Results for reinforcement bars:
 Distance between maximum compression corner of the section and neutral axis:
 height of a section: $h = 0.6$
 $y_b = -Eps_top \cdot h / (Eps_bot - Eps_top) = 0.0822604$
 $Eps_u1 = f_{yd} / E_s$

Concrete gross section gravity center:

$$x_{Cc} = 0.15$$

$$y_{Cc} = 0.30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar i:

$$\text{If } |eps_i| \leq f_{yd} / E_s$$

then

$$\sigma_i = \epsilon_i / \epsilon_{u1} \cdot f_{yd}$$

else

$$\sigma_i = f_{yd} + (|\epsilon_i| - \epsilon_{u1}) / (\epsilon_{u2} - \epsilon_{u1}) \cdot (k - 1) \cdot f_{yd}$$

i	x	y	ϕ	Distance from neutral axis	Strain $\epsilon = d / y_b \cdot \epsilon_{top}$	Stresses Sigma / 10e6	Force in bar $N = \sigma \cdot \pi \cdot \phi^2 / 4$	Positions in central axes coordinate system x_{cc} y_{cc}		$M_{xs} = -N \cdot y_{cc}$	$M_{ys} = N \cdot x_{cc}$
1	0,05	0,05	0,032	-0,468	-0,01990	-500	-402124	-0,10	0,25	100531	40212
2	0,05	0,55	0,032	0,032	0,00137	275	220784	-0,10	0,25	-55196	-22078
3	0,25	0,55	0,032	0,032	0,00137	275	220784	0,10	0,25	-55196	22078
4	0,25	0,05	0,032	-0,468	-0,01990	-500	-402124	0,10	0,25	100531	-40212
						Sum of forces in bars:	-362680			-311454	0

$$N_s = 362.68e3$$

$$M_{xs} = -311.45e3$$

$$M_{ys} = 0.00e3$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

i	x	y	ϕ	Positions in central axes coordinate system x_{cc} y_{cc}		Area of concrete with compression stress cut from the section in place of bars	Force N_{sc} in concrete in place of bars $N_{sc} = f_{cd} \cdot \epsilon_{top} / \epsilon_{ult} \cdot \text{Area}$	$M_{xsc} = -N_{sc} \cdot y_{cc}$	$M_{ysc} = N_{sc} \cdot x_{cc}$
1	0,05	0,05	0,032	-0,1	-0,25				
2	0,05	0,55	0,032	-0,1	0,25	0,000804248	16085	-4021	-1608
3	0,25	0,55	0,032	0,1	0,25	0,000804248	16085	-4021	1608
4	0,25	0,05	0,032	0,1	-0,25				
						Sum of forces :	32170	-8042	0

Reduction forces in concrete:

$$N_{sc} = 32.17e3$$

$$M_{xsc} = -8.04e3$$

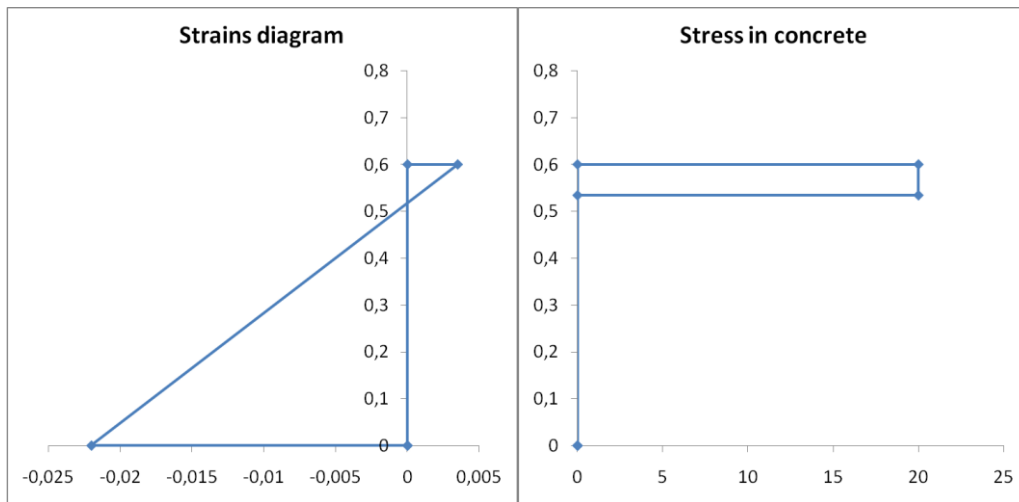
$$M_{ysc} = 0.00e3$$

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in Case 2a) in central axes coordinate system:

$$x_{scg} = \sum(x_{cc_i} \cdot \text{Area}_i) / \sum(\text{Area}_i) = 0.00$$

$$y_{scg} = \sum(y_{cc_i} \cdot \text{Area}_i) / \sum(\text{Area}_i) = 0.25$$

- Results for concrete section:



Height of compressed part of the section:

$$\text{Lambda} \cdot y_b = 0.065808351$$

Area of contour under compressive stress:

$$A_c = 0.3 \cdot 0.064608 = \underline{0.01974}$$

Stress in concrete:

$$\text{Sigma} = \text{Eps_top} / \text{Eps_ult} \cdot f_{cd} = 20\text{e}6$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = ((0.3 / 2 - x_{Cc}) \cdot A_c \cdot \text{Sigma} - x_{scg} \cdot N_{sc}) / (A_c \cdot \text{Sigma} + N_{sc}) = \underline{0}$$

$$y_{cg} = ((0.6 - \text{Lambda} \cdot y_b / 2 - y_{Cc}) \cdot A_c \cdot \text{Sigma} - y_{scg} \cdot N_{sc}) / (A_c \cdot \text{Sigma} + N_{sc}) = \underline{0.268612}$$

Internal forces in concrete:

$$N_c = A_c \cdot \text{Sigma} - N_{sc} = \underline{362.68\text{e}3}$$

$$M_{xc} = -N_c \cdot y_{cg} = \underline{-97.42\text{e}3}$$

$$M_{yc} = N_c \cdot x_{cg} = \underline{0.00\text{e}3}$$

- Results for reduced section:

$$N = N_c + N_s = \underline{0.00\text{e}3}$$

$$M_x = M_{xc} + M_{xs} = \underline{-408.87\text{e}3}$$

$$M_y = M_{yc} + M_{ys} = \underline{0.00\text{e}3}$$

3.5 Case 2c

3.5.1 Data:

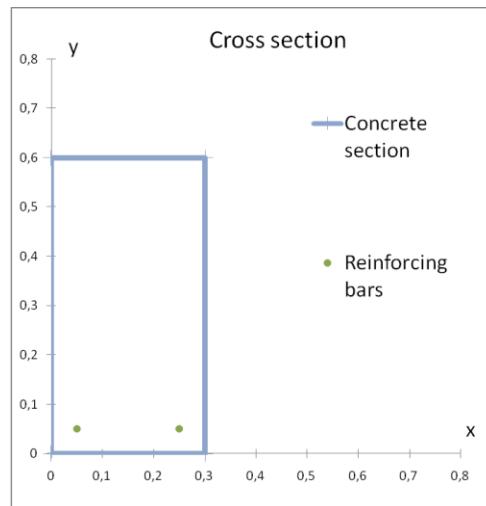
- Geometry:
 - Concrete section:

$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.6$

$x = 0.3$ $y = 0.6$
 $x = 0.3$ $y = 0.0$

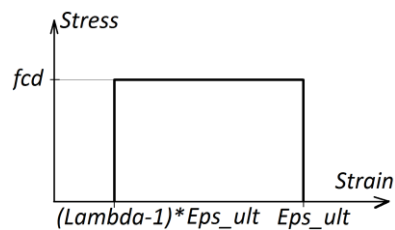
- Reinforcing bars:

$x = 0.05$ $y = 0.05$ $\phi = 0.032$
 $x = 0.25$ $y = 0.05$ $\phi = 0.032$



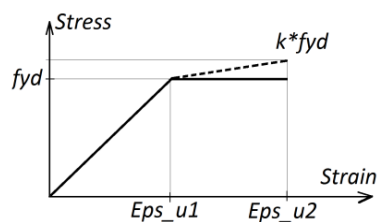
- Concrete parameters:

- Design strength $f_{cd} = 20e6$
- Effective height reduction factor $\Lambda = 0.8$
- Modulus of elasticity $E_c = 30e9$
- Strain-stress model: rectangular
- Strain ultimate limit $Eps_ult = 0.0035$

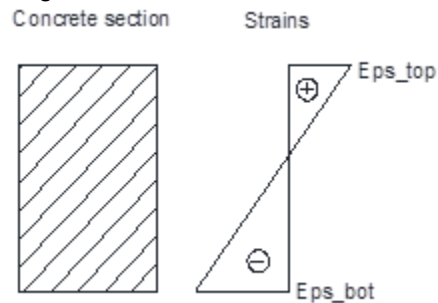


- Steel parameters:

- Design strength $f_{yd} = 500e6$
- Hardening factor $k = 1.0$
- Modulus of elasticity $E_s = 200e9$
- Strain ultimate limit $Eps_u2 = 0.075$



- Strains:
 - Top strain $Eps_top = 0.0015$
 - Bottom strain $Eps_bot = -0.002$
 - Neutral axis angle $Angle = \pi \cdot 3 / 2$



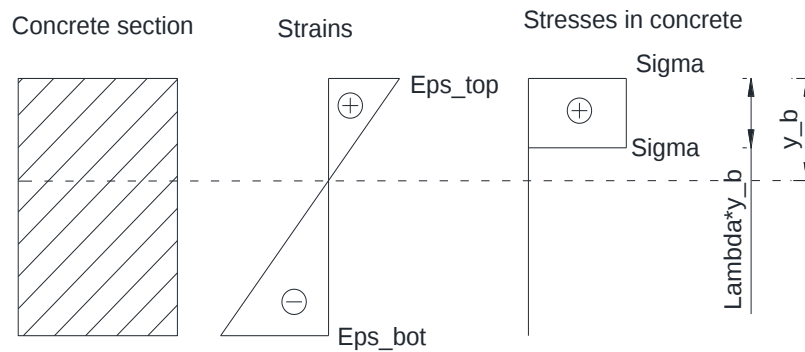
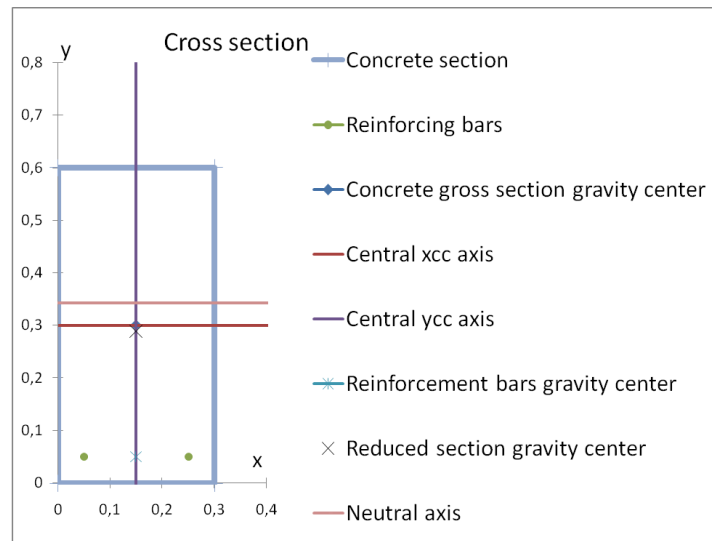
3.5.2 Search for:

- Results for reinforcement bars: N_s , M_{xs} , M_{ys}
- Results for concrete section: A_c , x_{cg} , y_{cg} , N_c , M_{xc} , M_{yc}
- Results for reduced section: N , M_x , M_y

3.5.3 Results from Component

- Results for reinforcement bars:
 - $N_s = -549.57e3$
 - $M_{xs} = -137.39e3$
 - $M_{ys} = 0.00e3$
- Results for concrete section:
 - $A_c = 0.0617$
 - $x_{cg} = 0.000000$
 - $y_{cg} = 0.197143$
 - $N_c = 528.98e3$
 - $M_{xc} = -104.28e3$
 - $M_{yc} = 0.00e3$
- Results for reduced section:
 - $N = -20.59e3$
 - $M_x = -241.68e3$
 - $M_y = 0.00e3$

3.5.4 Results (manual calculations):



- Results for reinforcement bars:
Distance between maximum compression corner of the section and neutral axis:

height of a section: $h = 0.6$

$$y_b = -Eps_top \cdot h / (Eps_bot - Eps_top) = 0.257143$$

$$Eps_u1 = f_{yd} / E_s$$

Concrete gross section gravity center:

$$x_{Cc} = 0.15$$

$$y_{Cc} = 0.30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar i:

$$\text{If } |eps_i| \leq f_{yd} / E_s \text{ then}$$

$$\sigma_i = eps_i / Eps_u1 \cdot f_{yd}$$

else

$$\sigma_i = f_{yd} + (|\epsilon_i| - \epsilon_{u1}) / (\epsilon_{u2} - \epsilon_{u1}) \cdot (k - 1) \cdot f_{yd}$$

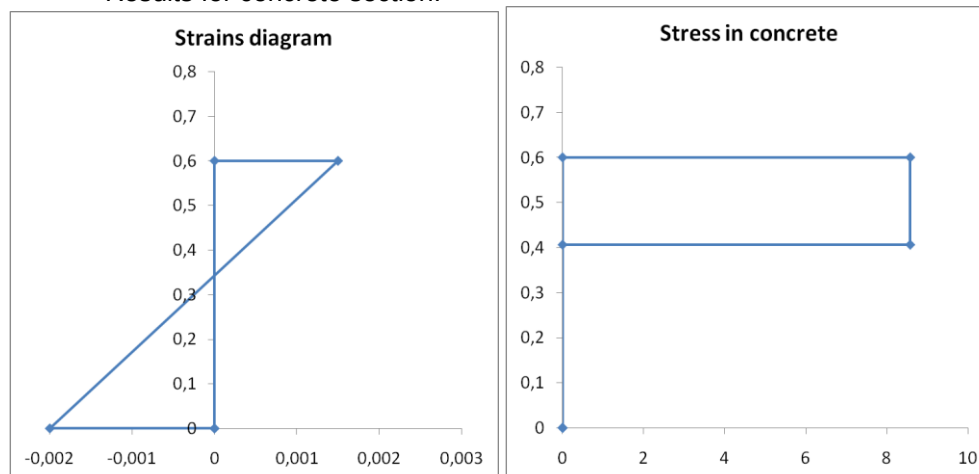
i	x	y	ϕ	Distance d from neutral axis	strain $\epsilon = d /$ $y_b \cdot$ ϵ_{u1}	stresses $\sigma /$ 10e6	force in bar $N =$ $\sigma \cdot \pi$ $\cdot \phi^2 / 4$	Positions in central axes coordinate system $x_{cc} y_{cc}$		$M_{xs} = -$ $N \cdot y_{cc}$	$M_{ys} = N$ $\cdot x_{cc}$
1	0,05	0,05	0,032	-0,293	-0,00171	-342	-274785	-0,10	-0,25	-68696	27478
2	0,25	0,05	0,032	-0,293	-0,00171	-342	-274785	0,10	-0,25	-68696	-27478
						Sum of forces in bars:	-549569			- 137392	0

$$N_s = -549.57e3$$

$$M_{xs} = -137.39e3$$

$$M_{ys} = 0.00e3$$

- Results for concrete section:



Height of compressed part of the section:

$$\lambda \cdot y_b = 0.205714$$

Area of contour under compressive stress:

$$A_c = 0.3 \cdot 0.205714 = 0.0617$$

Stress in concrete:

$$\sigma = \epsilon_{bot} / \epsilon_{ult} \cdot f_{cd} = 8.571e6$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = (0.3/2 - x_{cc}) \cdot A_c \cdot \sigma / (A_c \cdot \sigma) = 0$$

$$y_{cg} = (0.6 - \lambda \cdot y_b / 2 - y_{cc}) \cdot A_c \cdot \sigma / (A_c \cdot \sigma) = 0.197143$$

Internal forces in concrete:

$$N_c = A_c \cdot \sigma = 528.98e3$$

$$M_{xc} = -N_c \cdot y_{cg} = -104.28e3$$

$$M_{yc} = N_c \cdot x_{cg} = 0.00e3$$

- Results for reduced section:

$$N = N_c + N_s = -20.59e3$$

$$M_x = M_{xc} + M_{xs} = -241.68e3$$

$$M_y = M_{yc} + M_{ys} = \underline{0.00e3}$$

4 Case 3: Calculation of internal forces for given state of strain in the section and linear model of concrete

4.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, extreme strains in the section and give internal forces.

4.2 Summary:

This sample gives as results the internal forces for:

- A rectangular concrete shape section with symmetric reinforcement (Case 3a and Case 3b)
- A rectangular concrete shape section with asymmetric reinforcement (Case 3c)
- A linear model of concrete and horizontal branch model of steel
- A section under unidirectional bending with entire section under compression (Case 3a)
- A section under unidirectional bending with a part of section under compression (Case 3b and Case 3c)
- A state of strain with ultimate compressive strain in concrete (Case 3a and Case 3b)
- A strain below the ultimate compressive strain in concrete (Case 3c).

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel.

Based on input data, Component gives as output results:

- Results for reinforcement bars:
 - Axial force (N_s)
 - Moment with respect to x_{cc} axis (M_{xs})
 - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
 - Area of contour under compressive stress (A_c)
 - Center of gravity of stresses in the concrete (x_{cg} , y_{cg})
 - Axial force (N_c)
 - Moment with respect to x_{cc} axis (M_{xc})
 - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
 - Axial force (N)
 - Moment with respect to x_{cc} axis (M_x)
 - Moment with respect to y_{cc} axis (M_y)

For these results, x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

4.3 Case 3a

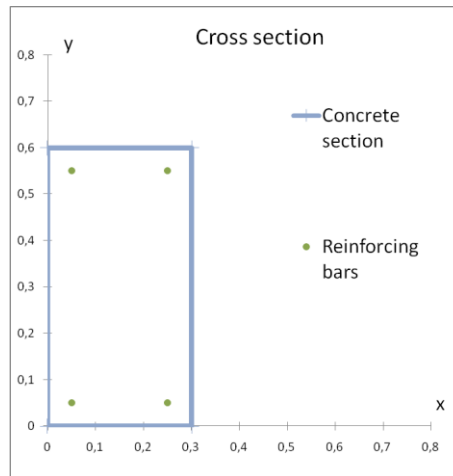
4.3.1 Data:

- Geometry:
 - Concrete section:

$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.6$
$x = 0.3$	$y = 0.6$
$x = 0.3$	$y = 0.0$

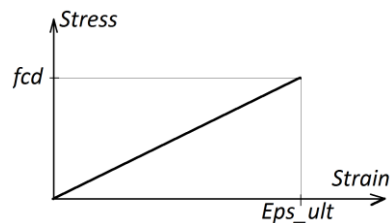
- Reinforcing bars:

$x = 0.05$	$y = 0.05$	$\phi = 0.012$
$x = 0.05$	$y = 0.55$	$\phi = 0.012$
$x = 0.25$	$y = 0.55$	$\phi = 0.012$
$x = 0.25$	$y = 0.05$	$\phi = 0.012$



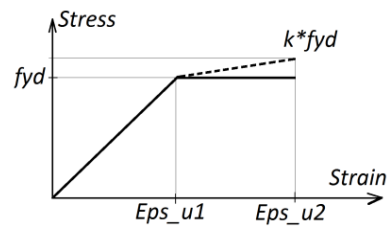
- Concrete parameters:

- Design strength $f_{cd} = 20e6$
- Modulus of elasticity $E_c = 30e9$
- Strain-stress model: linear
- Strain ultimate limit $Eps_ult = 0.0035$

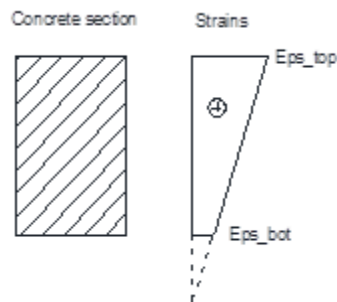


- Steel parameters:

- Design strength $f_{yd} = 500e6$
- Hardening factor $k = 1.0$
- Modulus of elasticity $E_s = 200e9$
- Strain ultimate limit $Eps_u2 = 0.075$



- Strains:
 - Top strain $Eps_top = 0.0035$
 - Bottom strain $Eps_bot = 0.0005$
 - Neutral axis angle $Angle = \pi \cdot 3 / 2$



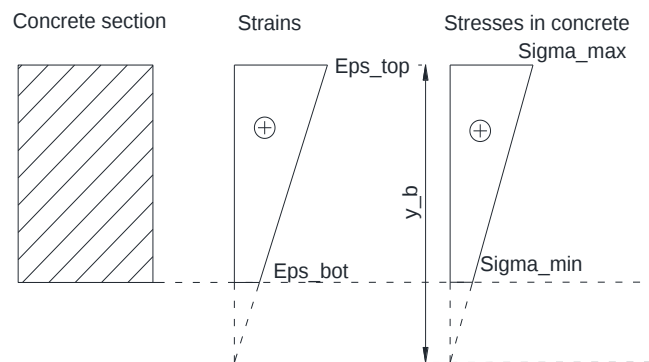
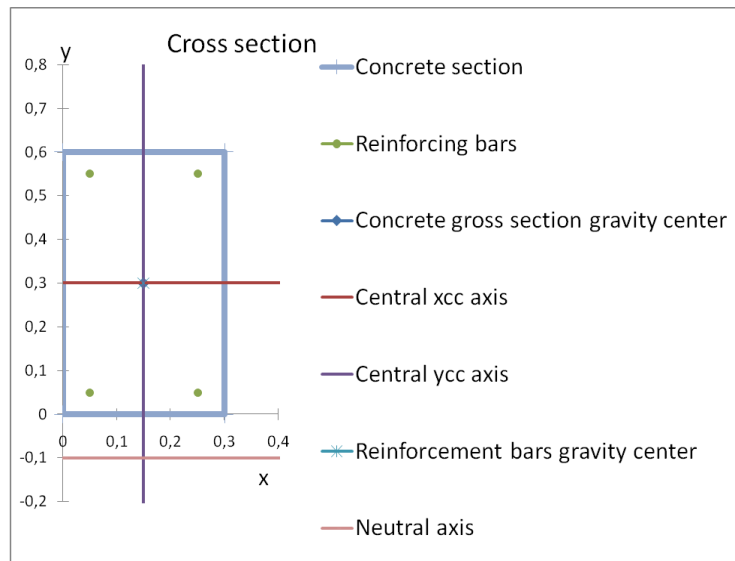
4.3.2 Search for:

- Results for reinforcement bars: N_s , M_{xs} , M_{ys}
- Results for concrete section: A_c , x_{cg} , y_{cg} , N_c , M_{xc} , M_{yc}
- Results for reduced section: N , M_x , M_y

4.3.3 Results from Component

- Results for reinforcement bars:
 - $N_s = 147.03e3$
 - $M_{xs} = -19.79e3$
 - $M_{ys} = 0.00e3$
- Results for concrete section:
 - $A_c = 0.1800$
 - $x_{cg} = 0.00000000$
 - $y_{cg} = 0.07479528$
 - $N_c = 2051.97e3$
 - $M_{xc} = -153.48e3$
 - $M_{yc} = 0.00e3$
- Results for reduced section:
 - $N = 2199.00e3$
 - $M_x = -173.27e3$
 - $M_y = 0.00e3$

4.3.4 Results (manual calculations):



- Results for reinforcement bars:
 Distance between maximum compression corner of the section and neutral axis:
 height of a section: $h = 0.6$
 $y_b = -Eps_top \cdot h / (Eps_bot - Eps_top) = 0.7$
 $Eps_u1 = f_{yd} / E_s$
 Concrete gross section gravity center:
 $x_{Cc} = 0.15$
 $y_{Cc} = 0.30$
 Distance between reinforcing bar and neutral axis:
 $d = y - h + y_b$
 Stress in bar i:
 If $|eps_i| \leq f_{yd} / E_s$ then
 $\sigma_i = eps_i / Eps_u1 \cdot f_{yd}$
 else
 $\sigma_i = f_{yd} + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) \cdot (k - 1) \cdot f_{yd}$

i	x	y	ϕ	Distance d from neutral axis	Strain eps = d / y_b • Eps_top	Stresses Sigma / 10e6	Force in bar N = sigma • $\pi \cdot \phi^2 / 4$	Positions in central axes coordinate system $x_{cc} y_{cc}$		$M_{xs} = -$ $N \cdot y_{cc}$	$M_{ys} = N$ $\cdot x_{cc}$
1	0,05	0,05	0,012	0,150	0,00075	150	16965	-0,1	-0,25	4241	-1696
2	0,05	0,55	0,012	0,650	0,00325	500	56549	-0,1	0,25	-14137	-5655
3	0,25	0,55	0,012	0,650	0,00325	500	56549	0,1	0,25	-14137	5655
4	0,25	0,05	0,012	0,150	0,00075	150	16965	0,1	-0,25	4241	1696
						Sum of forces in bars:	147027			-19792	0

$$N_s = 147.03e3$$

$$M_{xs} = -19.79e3$$

$$M_{ys} = 0.00e3$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

i	x	y	ϕ	Strain eps	Positions in central axes coordinate system $x_{cc} y_{cc}$		Area of concrete with compression stress cut from the section in place of bars	Force N_{sc} in concrete in place of bars $N_{sc} = f_{cd} \cdot$ eps / Eps_ult • Area	$M_{xsc} = -$ $N_{sc} \cdot y_{cc}$	$M_{ysc} =$ $N_{sc} \cdot$ x_{cc}
1	0,05	0,05	0,012	0,00075	-0,1	-0,25	0,000113097	485	121	-48
2	0,05	0,55	0,012	0,00325	-0,1	0,25	0,000113097	485	121	48
3	0,25	0,55	0,012	0,00325	0,1	0,25	0,000113097	2100	-525	210
4	0,25	0,05	0,012	0,00075	0,1	-0,25	0,000113097	2100	-525	-210
							Sum of forces:	5170	-808	0

Reduction forces in concrete:

$$N = 5.17e3$$

$$M_{xsc} = -0.81e3$$

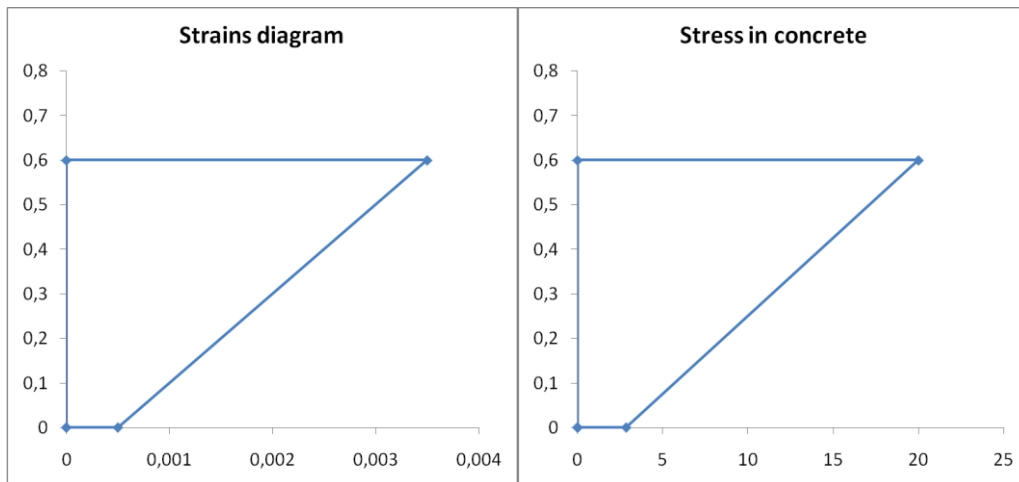
$$M_{ysc} = 0.000e3$$

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

$$x_{scg} = \text{sum}(x_{cc_i} \cdot \text{Area_i}) / \text{sum}(\text{Area_i}) = 0.00000$$

$$y_{scg} = \text{sum}(y_{cc_i} \cdot \text{Area_i}) / \text{sum}(\text{Area_i}) = 0.15625$$

- Results for concrete section:



Height of compressed part of the section:

$$0.6000$$

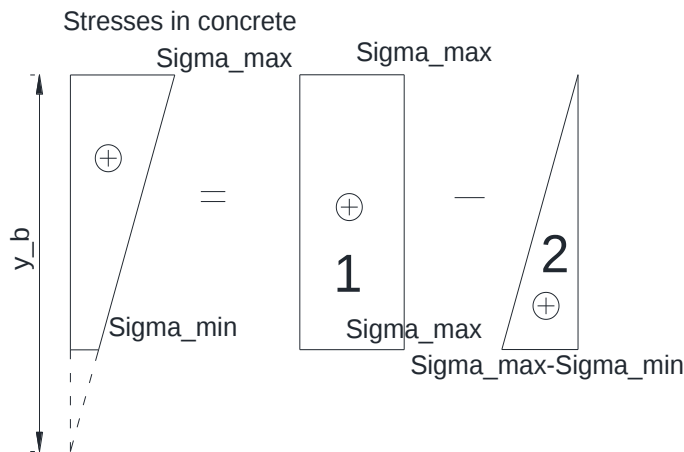
Area of contour under compressive stress:

$$A_c = 0.3 \cdot 0.6000 = \underline{0.1800}$$

Axial force N_c in compressed part of concrete section is calculated by subtraction of volume of a triangular solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of N_c in Introduction).

$$N_{c'} = N_{c1} - N_{c2}$$

$$N_c = N_{c'} - N_{sc}$$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

Moments M_{xc} and M_{yc} caused by stresses acting on entire compressed gross area are calculated following below equations:

$$M_{xc} = - N_c \cdot y_{cg}$$

$$M_{yc} = N_c \cdot x_{cg}$$

$$\text{Sigma_max} = f_{cd} \cdot \text{Eps_top} / \text{Eps_c1} = 20.000\text{e6}$$

$$\text{Sigma_min} = f_{cd} \cdot \text{Eps_bot} / \text{Eps_c1} = 2.857\text{e6}$$

Axial force in compressed part of concrete gross section

$$N_{c1} = A_c \cdot \text{Sigma_max} = 3.600\text{e6}$$

$$N_{c2} = A_c \cdot (\text{Sigma_max} - \text{Sigma_min}) / 2 = 1.543\text{e6}$$

$$N_{c'} = N_{c1} - N_{c2} = 2.057\text{e6}$$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$$x_{c1} = 0$$

$$y_{c1} = 0$$

$$x_{c2} = 0$$

$$y_{c2} = -0.3 + 0.6 / 3 = -0.1$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = \underline{0.000000}$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = \underline{0.074795}$$

Internal forces in concrete:

$$N_c = N_{c'} - N_{sc} = \underline{2051.97\text{e3}}$$

$$M_{xc} = -N_c \cdot y_{cg} = \underline{-153.48\text{e3}}$$

$$M_{yc} = N_c \cdot x_{cg} = \underline{0.00\text{e3}}$$

- Results for reduced section:

$$N = N_c + N_s = \underline{2199.00\text{e3}}$$

$$M_x = M_{xc} + M_{xs} = \underline{-173.27\text{e3}}$$

$$M_y = M_{yc} + M_{ys} = \underline{0.00\text{e3}}$$

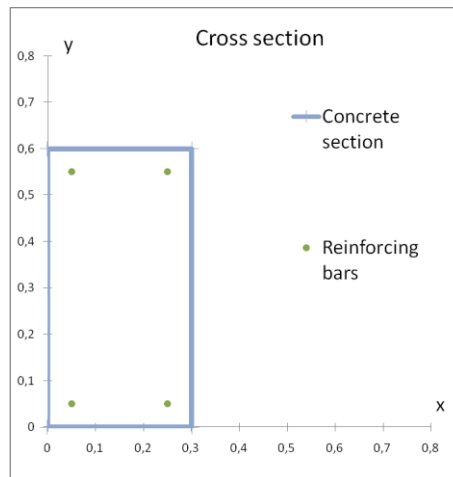
4.4 Case 3b

4.4.1 Data:

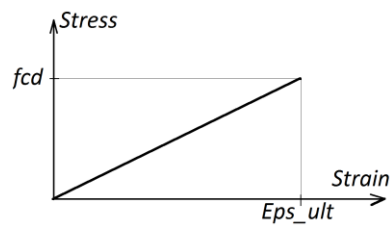
- Geometry:
 - Concrete section:

x = 0.0	y = 0.0
x = 0.0	y = 0.6
x = 0.3	y = 0.6
x = 0.3	y = 0.0
 - Reinforcing bars:

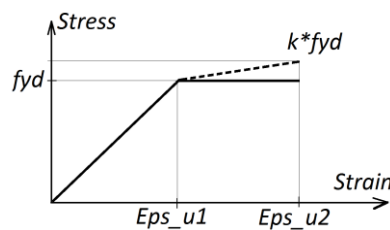
x = 0.05	y = 0.05	$\phi = 0.032$
x = 0.05	y = 0.55	$\phi = 0.032$
x = 0.25	y = 0.55	$\phi = 0.032$
x = 0.25	y = 0.05	$\phi = 0.032$



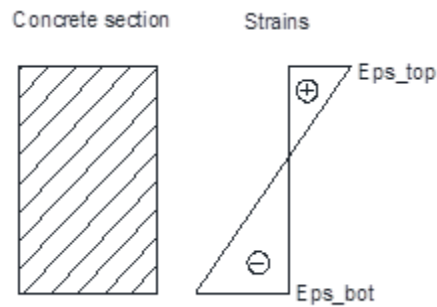
- Concrete parameters:
 - Design strength $f_{cd} = 20e6$
 - Modulus of elasticity $E_c = 30e9$
 - Strain-stress model: linear
 - Strain ultimate limit $Eps_ult = 0.0035$



- Steel parameters:
 - Design strength $f_{yd} = 500e6$
 - Hardening factor $k = 1.0$
 - Modulus of elasticity $E_s = 200e9$
 - Strain ultimate limit $Eps_u2 = 0.075$



- Strains:
 - Top strain $Eps_top = 0.0035$
 - Bottom strain $Eps_bot = - 0.01857592$
 - Neutral axis angle $Angle = \pi \cdot 3 / 2$



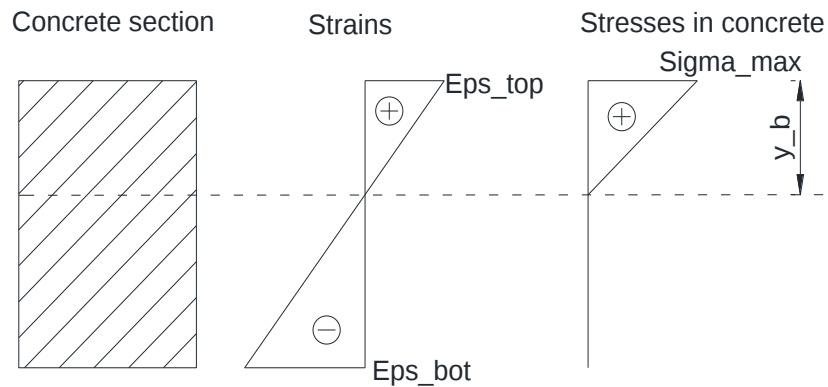
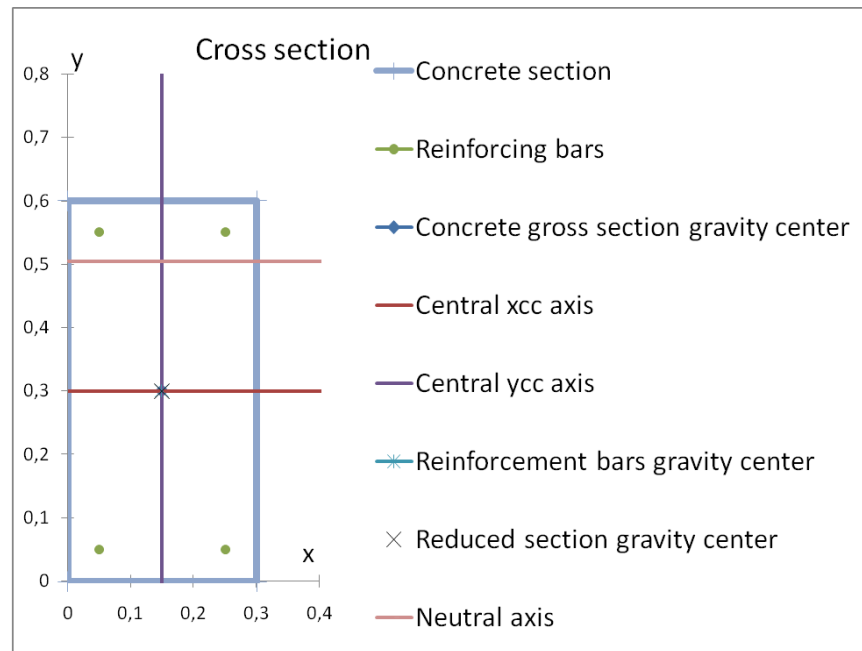
4.4.2 Search for:

- Results for reinforcement bars: N_s , M_{xs} , M_{ys}
- Results for concrete section: A_c , x_{cg} , y_{cg} , N_c , M_{xc} , M_{yc}
- Results for reduced section: N , M_x , M_y

4.4.3 Results from Component

- Results for reinforcement bars:
 $N_s = -270.12e3$
 $M_{xs} = -334.59e3$
 $M_{ys} = 0.00e3$
- Results for concrete section:
 $A_c = 0.0285$
 $x_{cg} = 0.00000000$
 $y_{cg} = 0.26932464$
 $N_c = 270.12e3$
 $M_{xc} = -72.75e3$
 $M_{yc} = 0.00e3$
- Results for reduced section:
 $N = 0.00e3$
 $M_x = -407.34e3$
 $M_y = 0.00e3$

4.4.4 Results (manual calculations):



- Results for reinforcement bars:
 Distance between maximum compression corner of the section and neutral axis:
 height of a section: $h = 0.6$
 $y_b = - \epsilon_{top} \cdot h / (\epsilon_{bot} - \epsilon_{top}) = 0.095126$
 $\epsilon_{u1} = f_{yd} / E_s$

Concrete gross section gravity center:

$$x_{Cc} = 0.15$$

$$y_{Cc} = 0.30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar i:

i	x	y	ϕ	Distance d from neutral axis	Strain $\epsilon_s = d / y_b \cdot \epsilon_{s_top}$	Stresses $\sigma / 10e6$	Force in bar $N = \sigma \cdot \pi \cdot \phi^2 / 4$	Positions in central axes coordinate system x_{cc} y_{cc}		$M_{xs} = -N \cdot y_{cc}$	$M_{ys} = N \cdot x_{cc}$
1	0,05	0,05	0,032	-0,455	-0,01674	-500	-402124	-0,10	-0,25	100531	40212
2	0,25	0,05	0,032	-0,455	-0,01674	-500	-402124	0,10	-0,25	100531	-40212
3	0,25	0,55	0,032	0,045	0,00166	332	267065	0,10	0,25	-66766	26706
4	0,05	0,55	0,032	0,045	0,00166	332	267065	-0,10	0,25	-66766	-26706
						Sum of forces in bars:	-270118			334594	0

If $|\epsilon_{s_i}| \leq f_{yd} / E_s$ then

$$\sigma_i = \epsilon_{s_i} / \epsilon_{s_u1} \cdot f_{yd}$$

else

$$\sigma_i = f_{yd} + (|\epsilon_{s_i}| - \epsilon_{s_u1}) / (\epsilon_{s_u2} - \epsilon_{s_u1}) \cdot (k - 1) \cdot f_{yd}$$

$$N_s = -270.12e3$$

$$M_{xs} = -334.594e3$$

$$M_{ys} = 0.00e3$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

i	x	y	ϕ	Strain ϵ_s	Positions in central axes coordinate system x_{cc} y_{cc}		Area of concrete with compression stress cut from the section in place of bars	Force N_{sc} in concrete in place of bars $N_{sc} = f_{cd} \cdot \epsilon_s / \epsilon_{s_ult} \cdot \text{Area}$	$M_{xsc} = -N_{sc} \cdot y_{cc}$	$M_{ysc} = N_{sc} \cdot x_{cc}$
1	0,05	0,05	0,032	-0,01674	-0,1	0,25				
2	0,25	0,05	0,032	-0,01674	0,1	0,25				
3	0,25	0,55	0,032	0,00166	0,1	0,25	0,000804248	7630	-1908	763
4	0,05	0,55	0,032	0,00166	-0,1	0,25	0,000804248	7630	-1908	-763
							Sum of forces :	15261	-3815	0

Reduction forces in concrete:

$$N_{sc} = 15.26e3$$

$$M_{xsc} = -3.82e3$$

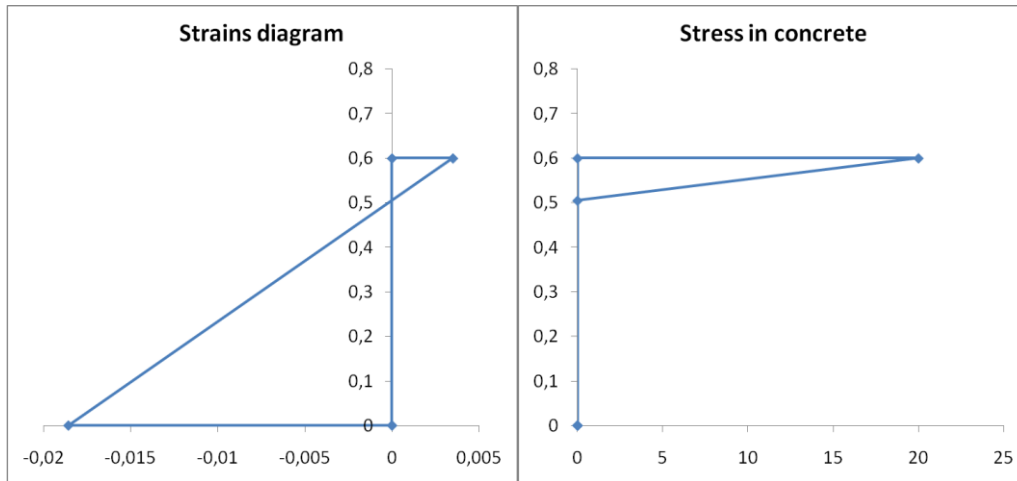
$$M_{ysc} = 0.00e3$$

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

$$x_{scg} = \sum(x_{cc_i} \cdot \text{Area}_i) / \sum(\text{Area}_i) = 0.00$$

$$y_{scg} = \sum(y_{cc_i} \cdot \text{Area}_i) / \sum(\text{Area}_i) = 0.25$$

- Results for concrete section:



Height of compressed part of the section:

$$y_b = 0.095126$$

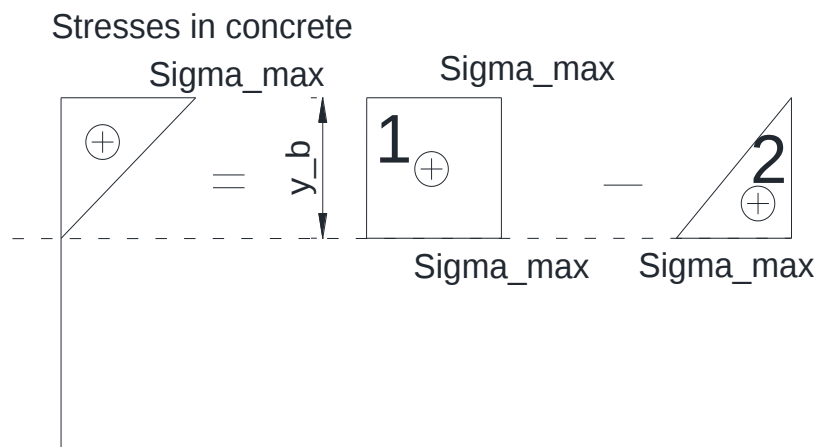
Area of contour under compressive stress:

$$A_c = 0.3 \cdot y_b = 0.0285$$

Axial force N_c in compressed part of concrete section is calculated by subtraction of volume of a triangular solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of N_c in Introduction).

$$N_{c'} = N_{c1} - N_{c2}$$

$$N_c = N_{c'} - N_{sc}$$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

Moments M_{xc} and M_{yc} caused by stresses acting on entire compressed gross area are calculated following below equations:

$$M_{xc} = - N_c \cdot y_{cg}$$

$$M_{yc} = N_c \cdot x_{cg}$$

$$\text{Sigma_max} = f_{cd} \cdot \text{Eps_top} / \text{Eps_ult} = 20.000\text{e}6$$

Axial force in compressed part of concrete gross section

$$N_{c1} = A_c \cdot \text{Sigma_max} = 0.5708\text{e}6$$

$$N_{c2} = A_c \cdot \text{Sigma_max} / 2 = 0.2854\text{e}6$$

$$N_{c'} = N_{c1} - N_{c2} = 0.28538\text{e}6$$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$$x_{c1} = 0$$

$$y_{c1} = 0.3 - y_b / 2 = 0.25243687$$

$$x_{c2} = 0$$

$$y_{c2} = 0.3 - y_b \cdot 2/3 = 0.23658249$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.00000000$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.26932464$$

Internal forces in concrete:

$$N_c = N_{c'} - N_{sc} = 270.12\text{e}3$$

$$M_{xc} = -N_c \cdot y_{cg} = -72.75\text{e}3$$

$$M_{yc} = N_{c'} \cdot x_{cg} = 0.00\text{e}3$$

- Results for reduced section:

$$N = N_c + N_s = 0.00\text{e}3$$

$$M_x = M_{xc} + M_{xs} = -407.34\text{e}3$$

$$M_y = M_{yc} + M_{ys} = 0.00\text{e}3$$

4.5 Case 3c

4.5.1 Data:

- Geometry:

- Concrete section:

$$x = 0.0 \quad y = 0.0$$

$$x = 0.0 \quad y = 0.6$$

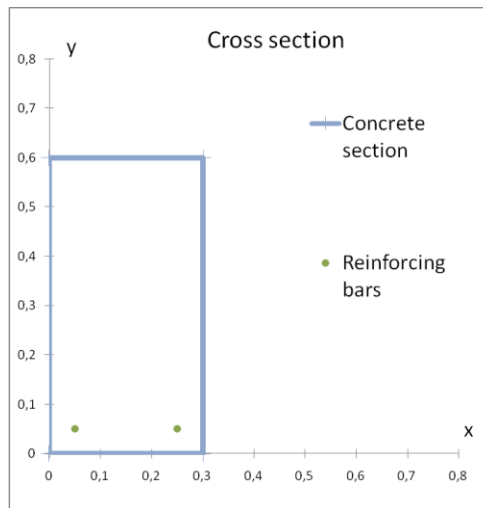
$$x = 0.3 \quad y = 0.6$$

$$x = 0.3 \quad y = 0.0$$

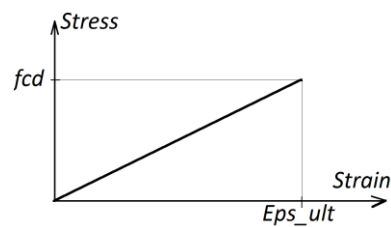
- Reinforcing bars:

$$x = 0.05 \quad y = 0.05 \quad \phi = 0.032$$

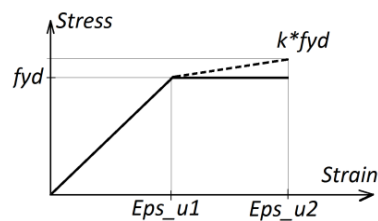
$$x = 0.25 \quad y = 0.05 \quad \phi = 0.032$$



- Concrete parameters:
 - Design strength $f_{cd} = 20e6$
 - Modulus of elasticity $E_c = 30e9$
 - Strain-stress model: linear
 - Strain ultimate limit $Eps_ult = 0.0035$



- Steel parameters:
 - Design strength $f_{yd} = 500e6$
 - Hardening factor $k = 1.0$
 - Modulus of elasticity $E_s = 200e9$
 - Strain ultimate limit $Eps_u2 = 0.075$

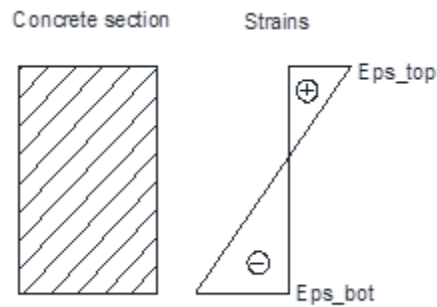


Strains:

Top strain $Eps_top = 0.0015$

Bottom strain $Eps_bot = -0.002$

Neutral axis angle $Angle = \pi \cdot 3 / 2$



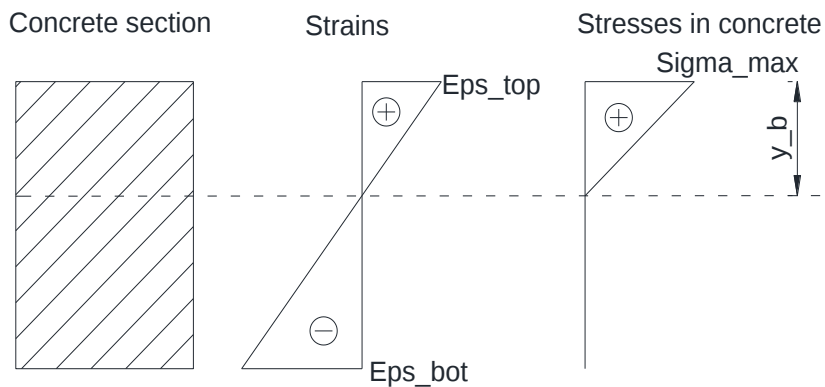
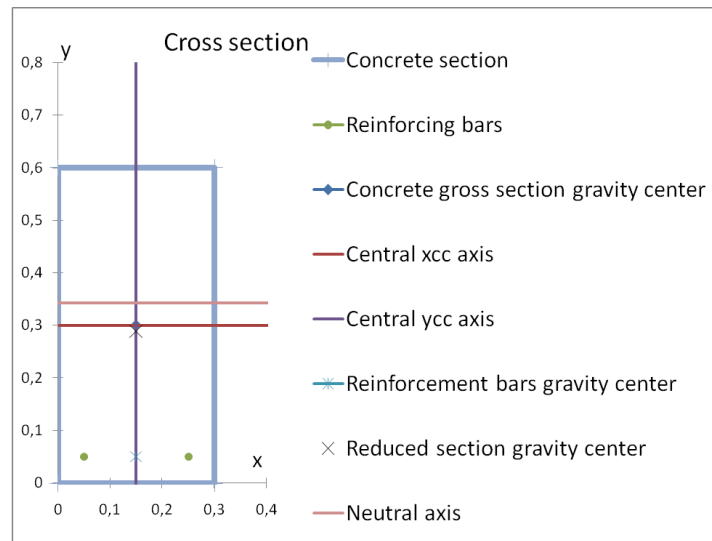
4.5.2 Search for:

- Results for reinforcement bars: N_s , M_{xs} , M_{ys}
- Results for concrete section: A_c , x_{cg} , y_{cg} , N_c , M_{xc} , M_{yc}
- Results for reduced section: N , M_x , M_y

4.5.3 Results from Component

- Results for reinforcement bars:
 $N_s = -549.57e3$
 $M_{xs} = -137.39e3$
 $M_{ys} = 0.00e3$
- Results for concrete section:
 $A_c = 0.0771$
 $x_{cg} = 0.000000$
 $y_{cg} = 0.21428571$
 $N_c = 330.61e3$
 $M_{xc} = -70.85e3$
 $M_{yc} = 0.00e3$
- Results for reduced section:
 $N = -218.96e3$
 $M_x = -208.24e3$
 $M_y = 0.00e3$

4.5.4 Results (manual calculations):



- Results for reinforcement bars:
 Distance between maximum compression corner of the section and neutral axis:
 height of a section: $h = 0.6$
 $y_b = -Eps_top \cdot h / (Eps_bot - Eps_top) = 0.257143$
 $Eps_u1 = f_{yd} / E_s$

Concrete gross section gravity center:
 $x_{cc} = 0.15$
 $y_{cc} = 0.30$

Distance between reinforcing bar and neutral axis:
 $d = y - h + y_b$

Stress in bar i:
 If $|eps_i| \leq f_{yd} / E_s$ then
 $\sigma_i = eps_i / Eps_u1 \cdot f_{yd}$
 else
 $\sigma_i = f_{yd} + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) \cdot (k - 1) \cdot f_{yd}$

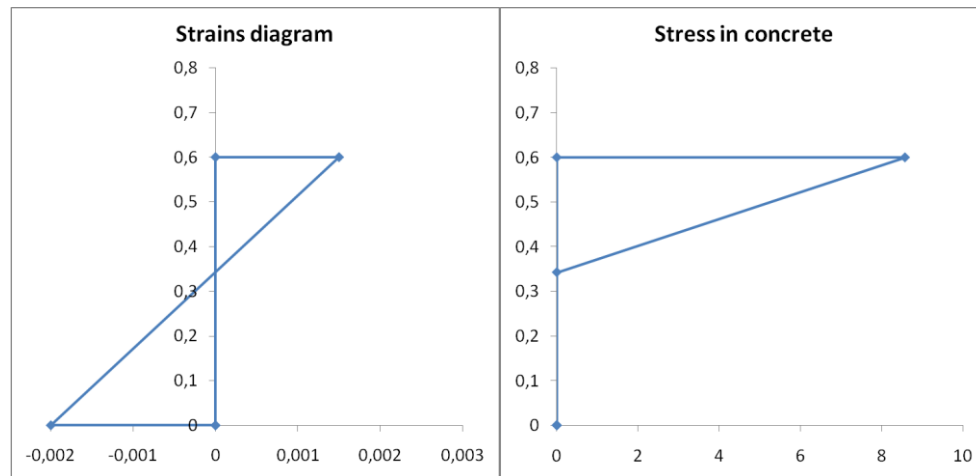
i	x	y	ϕ	Distance d from neutral axis	Strain $\epsilon = d / y_b \cdot \epsilon_{top}$	Stresses $\sigma / 10e6$	force in bar $N = \sigma \cdot \pi \cdot \phi^2 / 4$	Positions in central axes coordinate system x_{cc} y_{cc}		$M_{xs} = -N \cdot y_{cc}$	$M_{ys} = N \cdot x_{cc}$
1	0,05	0,05	0,032	-0,293	-0,00171	-342	-274785	0,10	0,25	-68696	27478
2	0,25	0,05	0,032	-0,293	-0,00171	-342	-274785	0,10	0,25	-68696	-27478
Sum of forces in bars:							-549569			137392	0

$$N_s = -549.57e3$$

$$M_{xs} = -137.39e3$$

$$M_{ys} = 0.00e3$$

- Results for concrete section:



Height of compressed part of the section:

$$y_b = 0.257143$$

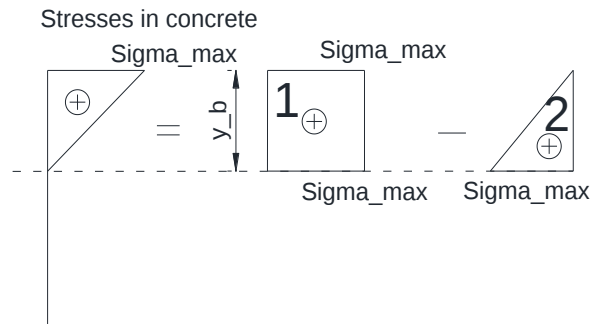
Area of contour under compressive stress:

$$A_c = 0.3 \cdot y_b = 0.0771$$

Axial force N_c in compressed part of concrete section is calculated by subtraction of volume of a triangular solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of N_c in Introduction).

$$N_c' = N_{c1} - N_{c2}$$

$$N_c = N_c'$$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2})$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2})$$

Moments M_{xc} and M_{yc} caused by stresses acting on entire compressed gross area are calculated following below equations:

$$M_{xc} = - N_c \cdot y_{cg}$$

$$M_{yc} = N_c \cdot x_{cg}$$

$$\text{Sigma_max} = f_{cd} \cdot \text{Eps_top} / \text{Eps_ult} = 8.571\text{e}6$$

Axial force in compressed part of concrete gross section

$$N_{c1} = A_c \cdot \text{Sigma_max} = 0.6612\text{e}6$$

$$N_{c2} = A_c \cdot \text{Sigma_max} / 2 = 0.3306\text{e}6$$

$$N_{c'} = N_{c1} - N_{c2} = 0.3306\text{e}6$$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$$x_{c1} = 0$$

$$y_{c1} = 0.3 - y_b / 2 = 0.17142857$$

$$x_{c2} = 0$$

$$y_{c2} = 0.3 - y_b \cdot 2/3 = 0.12857143$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2}) = 0.00000000$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2}) = 0.21428571$$

Internal forces in concrete:

$$N_c = N_{c'} = 330.61\text{e}3$$

$$M_{xc} = - N_c \cdot y_{cg} = -70.85\text{e}3$$

$$M_{yc} = N_c \cdot x_{cg} = 0.00\text{e}3$$

- Results for reduced section:

$$N = N_c + N_s = -218.96\text{e}3$$

$$M_x = M_{xc} + M_{xs} = -208.24\text{e}3$$

$$M_y = M_{yc} + M_{ys} = 0.00\text{e}3$$

5 Case 4: Calculation of internal forces for given state of strain in the section and bilinear model of concrete

5.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, extreme strains in the section and give internal forces.

5.2 Summary:

This sample gives as results the internal forces for:

- A rectangular concrete shape section with symmetric reinforcement (Case 4a and Case 4b)
- A rectangular concrete shape section with asymmetric reinforcement (Case 4c)
- A bilinear model of concrete and horizontal branch model of steel
- A section under unidirectional bending with entire section under compression (Case 4a)
- A section under unidirectional bending with a part of section under compression (Case 4b and Case 4c)
- A state of strain with ultimate compressive strain in concrete (Case 4a and Case 4b)
- A strain below the ultimate compressive strain in concrete (Case 4c).

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel.

Based on input data, Component gives as output results:

- Results for reinforcement bars:
 - Axial force (N_s)
 - Moment with respect to x_{cc} axis (M_{xs})
 - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
 - Area of contour under compressive stress (A_c)
 - Center of gravity of stresses in the concrete (x_{cg} , y_{cg})
 - Axial force (N_c)
 - Moment with respect to x_{cc} axis (M_{xc})
 - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
 - Axial force (N)
 - Moment with respect to x_{cc} axis (M_x)
 - Moment with respect to y_{cc} axis (M_y)

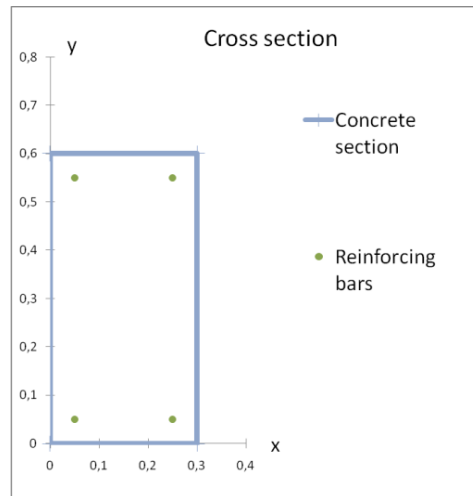
For these results, x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

5.3 Case 4a

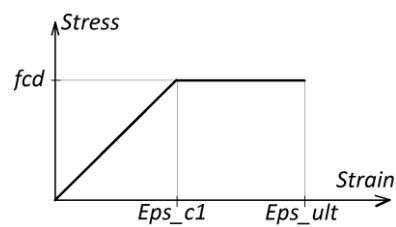
5.3.1 Data:

- Geometry:

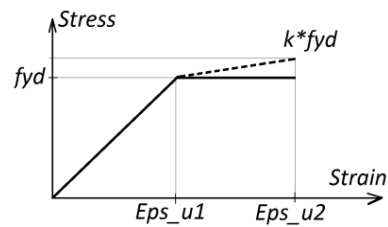
- Concrete section:
 - $x = 0.0$ $y = 0.0$
 - $x = 0.0$ $y = 0.6$
 - $x = 0.3$ $y = 0.6$
 - $x = 0.3$ $y = 0.0$
- Reinforcing bars:
 - $x = 0.05$ $y = 0.05$ $\phi = 0.016$
 - $x = 0.25$ $y = 0.05$ $\phi = 0.016$
 - $x = 0.25$ $y = 0.55$ $\phi = 0.016$
 - $x = 0.05$ $y = 0.55$ $\phi = 0.016$



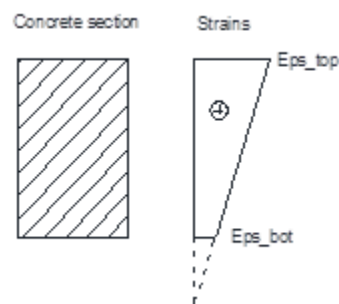
- Concrete parameters:
 - Design strength $f_{cd} = 30e6$
 - Modulus of elasticity $E_c = 32e9$
 - Strain-stress model: bilinear
 - Strain ultimate limit $Eps_ult = 0.0035$
 - Strain relation change over $Eps_c1 = 0.0020$



- Steel parameters:
 - Design strength $f_{yd} = 400e6$
 - Hardening factor $k = 1.0$
 - Modulus of elasticity $E_s = 205e9$
 - Strain ultimate limit $Eps_u2 = 0.1$



- Strains:
 - Top strain $Eps_top = 0.0035$
 - Bottom strain $Eps_bot = 0.0005$
 - Neutral axis angle $Angle = \pi \cdot 3 / 2$



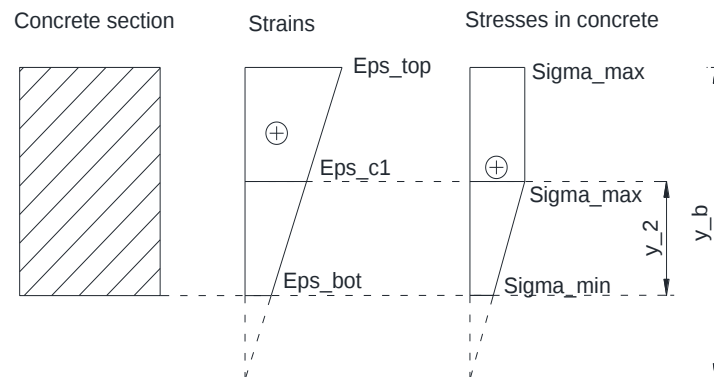
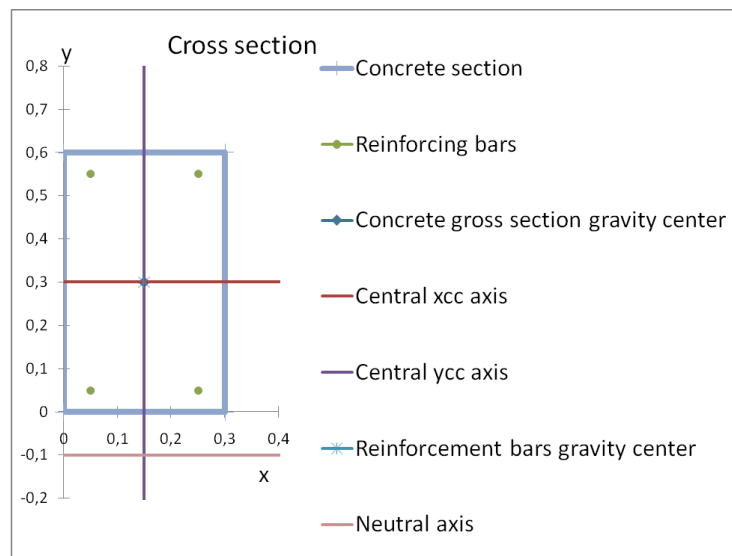
5.3.2 Search for:

- Results for reinforcement bars: N_s , M_{xs} , M_{ys}
- Results for concrete section: A_c , x_{cg} , y_{cg} , N_c , M_{xc} , M_{yc}
- Results for reduced section: N , M_x , M_y

5.3.3 Results from Component

- Results for reinforcement bars:
 - $N_s = 222.68e3$
 - $M_{xs} = -24.76e3$
 - $M_{ys} = 0.00e3$
- Results for concrete section:
 - $A_c = 0.1800$
 - $x_{cg} = 0.00000000$
 - $y_{cg} = 0.04589775$
 - $N_c = 4370.91e3$
 - $M_{xc} = -200.62e3$
 - $M_{yc} = 0.00e3$
- Results for reduced section:
 - $N = 4593.59e3$
 - $M_x = -225.37e3$
 - $M_y = 0.00e3$

5.3.4 Results (manual calculations):



- Results for reinforcement bars:

Distance between maximum compression corner of the section and neutral axis:

height of a section: $h = 0.6$

$$y_b = -Eps_top \cdot h / (Eps_bot - Eps_top) = 0.7$$

$$Eps_u1 = f_{yd} / E_s$$

Concrete gross section gravity center:

$$x_{Cc} = 0.15$$

$$y_{Cc} = 0.30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar i:

$$\text{If } |eps_i| \leq f_{yd} / E_s \text{ then}$$

$$\sigma_i = \epsilon_i / \epsilon_{u1} \cdot f_{yd}$$

else

$$\sigma_i = f_{yd} + (|\epsilon_i| - \epsilon_{u1}) / (\epsilon_{u2} - \epsilon_{u1}) \cdot (k - 1) \cdot f_{yd}$$

i	x	y	ϕ	Distance d from neutral axis	strain $\epsilon = d /$ $y_b \cdot$ ϵ_{top}	stresses $\sigma /$ 10e6	force in bar $N =$ $\sigma \cdot \pi$ $\cdot \phi^2 / 4$	Positions in central axes coordinate system x_{cc} y_{cc}		$M_{xs} = -N$ $\cdot y_{cc}$	$M_{ys} = N$ $\cdot x_{cc}$
1	0,05	0,05	0,016	0,150	0,00075	154	30913	-0,10	-0,25	7728	-3091
2	0,25	0,05	0,016	0,150	0,00075	154	30913	0,10	-0,25	7728	3091
3	0,25	0,55	0,016	0,650	0,00325	400	80425	0,10	0,25	-20106	8042
4	0,05	0,55	0,016	0,650	0,00325	400	80425	-0,10	0,25	-20106	-8042
						Sum of forces in bars:	222676			-24756	0

$$N_s = 222.68e3$$

$$M_{xs} = -24.76e3$$

$$M_{ys} = 0.00e3$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

$$\text{if } \epsilon \leq \epsilon_{c1} \text{ then } N_{sc} = f_{cd} \cdot \epsilon / \epsilon_{c1} \cdot \text{Area}$$

$$\text{if } \epsilon > \epsilon_{c1} \text{ then } N_{sc} = f_{cd} \cdot \text{Area}$$

i	x	y	ϕ	strain ϵ	Positions in central axes coordinate system x_{cc} y_{cc}		Area of concrete with compression stress cut from the section in place of bars	Force in concrete in place of bars N_{sc}	$M_{xsc} = -$ $N_{sc} \cdot y_{cc}$	$M_{ysc} =$ $N_{sc} \cdot x_{cc}$
1	0,05	0,05	0,016	0,00075	-0,1	-0,25	0,000201062	2262	565	-226
2	0,25	0,05	0,016	0,00075	0,1	-0,25	0,000201062	2262	565	226
3	0,25	0,55	0,016	0,00325	0,1	0,25	0,000201062	6032	-1508	603
4	0,05	0,55	0,016	0,00325	-0,1	0,25	0,000201062	6032	-1508	-603
							Sum of forces:	16588	-1885	0

Reduction forces in concrete:

$$N_{sc} = 16.59e3$$

$$M_{xsc} = -1.89e3$$

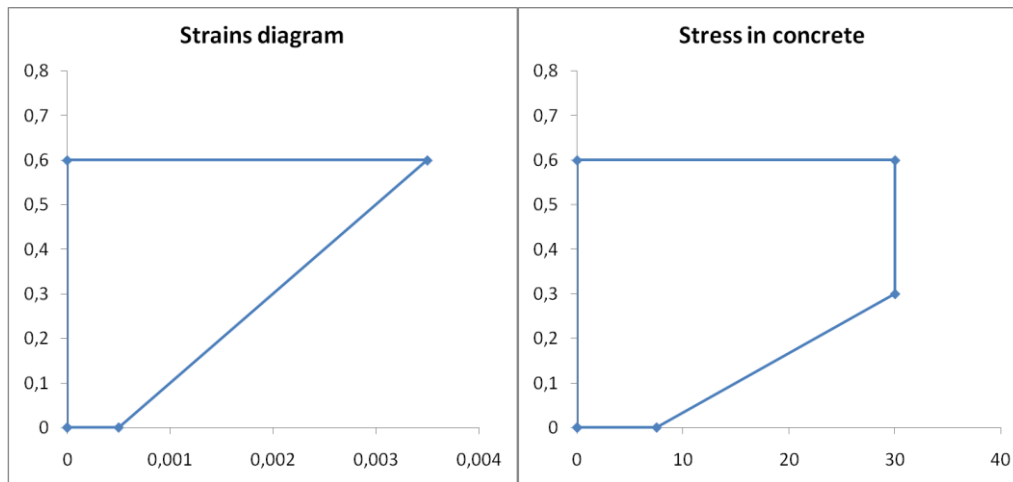
$$M_{ysc} = 0.00e3$$

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

$$x_{scg} = \sum(x_{cc_i} \cdot \text{Area_i}) / \sum(\text{Area_i}) = 0.00000$$

$$y_{scg} = \sum(y_{cc_i} \cdot \text{Area_i}) / \sum(\text{Area_i}) = 0.11364$$

- Results for concrete section:



Height of compressed part of the section:

$$h = 0.6000$$

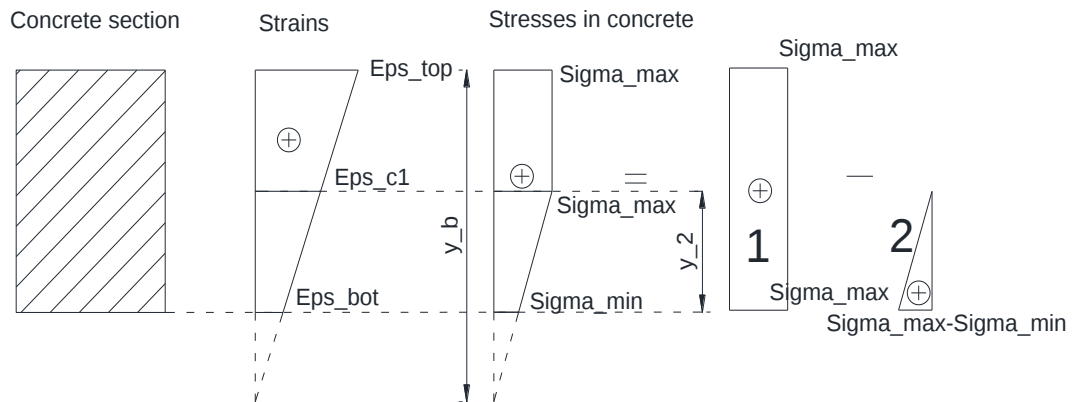
Area of contour under compressive stress:

$$A_c = 0.3 \cdot 0.6000 = 0.1800$$

Axial force N_c in compressed part of concrete section is calculated by subtraction of volume of a triangular solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of N_c in Introduction).

$$N_{c'} = N_{c1} - N_{c2}$$

$$N_c = N_{c'} - N_{sc}$$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

Moments M_{xc} and M_{yc} caused by stresses acting on entire compressed gross area are calculated following below equations:

$$M_{xc} = - N_c \cdot y_{cg}$$

$$M_{yc} = N_c \cdot x_{cg}$$

$$\text{Sigma_max} = f_{cd} = 30.000e6$$

$$\text{Sigma_min} = f_{cd} \cdot \text{Eps_bot} / \text{Eps_c1} = 7.500\text{e6}$$

Distance between gravity center of contour and neutral axis:

$$\text{delta} = - (0.6 / 2 - y_b) = 0.4$$

$$\text{Angle} = 3 / 2 \cdot \pi = 4.712$$

Position of axis with stresses equal Eps_c1 in central axes coordinate system:

$$\text{delta2} = - \text{delta} + \text{MIN}(y_b; \text{Eps_c1} / \text{Eps_top} \cdot y_b) = 0$$

$$\text{Angle} = 3 / 2 \cdot \pi = 4.712$$

Height of the part of the section where concrete stress is not constant:

$$y_2 = 0.3 - \text{delta2} = 0.3$$

Axial force in compressed part of concrete gross section

$$N_{c1} = A_c \cdot \text{Sigma_max} = 5.400\text{e6}$$

$$N_{c2} = 0.3 \cdot y_2 \cdot (\text{Sigma_max} - \text{Sigma_min}) / 2 = 1.0125\text{e6}$$

$$N_c = N_{c1} - N_{c2} = 4.3875\text{e6}$$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$$x_{c1} = 0$$

$$y_{c1} = 0$$

$$x_{c2} = 0$$

$$y_{c2} = - 0.3 + y_2 / 3 = - 0.2$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.000000$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.045898$$

Internal forces in concrete:

$$N_c = N_{c'} - N_{sc} = 4370.91\text{e3}$$

$$M_{xc} = - N_c \cdot y_{cg} = -200.62\text{e3}$$

$$M_{yc} = N_c \cdot x_{cg} = 0.00\text{e3}$$

- Results for reduced section:

$$N = N_c + N_s = 4593.59\text{e3}$$

$$M_x = M_{xc} + M_{xs} = -225.37\text{e3}$$

$$M_y = M_{yc} + M_{ys} = 0.00\text{e3}$$

5.4 Case 4b

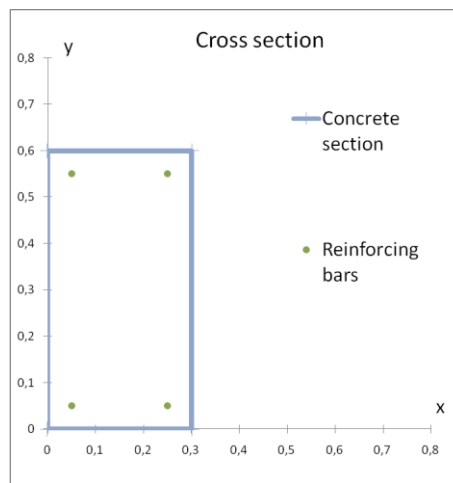
5.4.1 Data:

- Geometry:
 - Concrete section:

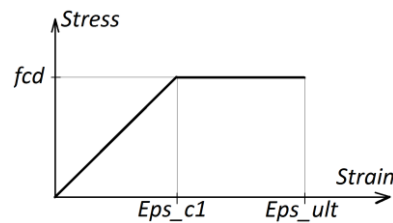
x = 0.0	y = 0.0
x = 0.0	y = 0.6
x = 0.3	y = 0.6
x = 0.3	y = 0.0
 - Reinforcing bars:

x = 0.05	y = 0.05	φ = 0.032
x = 0.05	y = 0.55	φ = 0.032
x = 0.25	y = 0.55	φ = 0.032

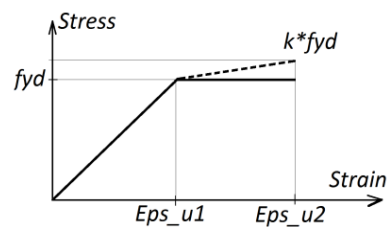
$$x = 0.25 \quad y = 0.05 \quad \phi = 0.032$$



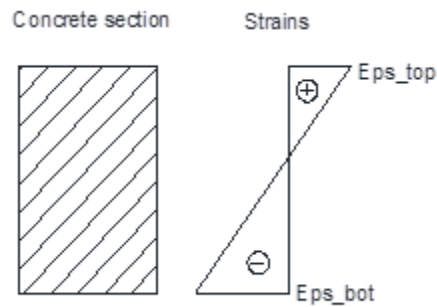
- Concrete parameters:
 - Design strength $f_{cd} = 30e6$
 - Modulus of elasticity $E_c = 32e9$
 - Strain-stress model: bilinear
 - Strain ultimate limit $Eps_ult = 0.0035$
 - Strain relation change over $Eps_c1 = 0.0020$



- Steel parameters:
 - Design strength $f_{yd} = 400e6$
 - Hardening factor $k = 1.0$
 - Modulus of elasticity $E_s = 205e9$
 - Strain ultimate limit $Eps_u2 = 0.1$



- Strains:
 - Top strain $Eps_top = 0.0035$
 - Bottom strain $Eps_bot = -0.02936558$
 - Neutral axis angle $Angle = \pi \cdot 3 / 2$



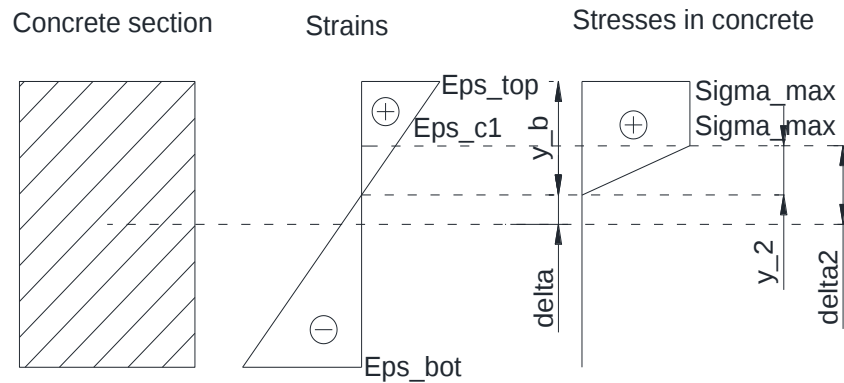
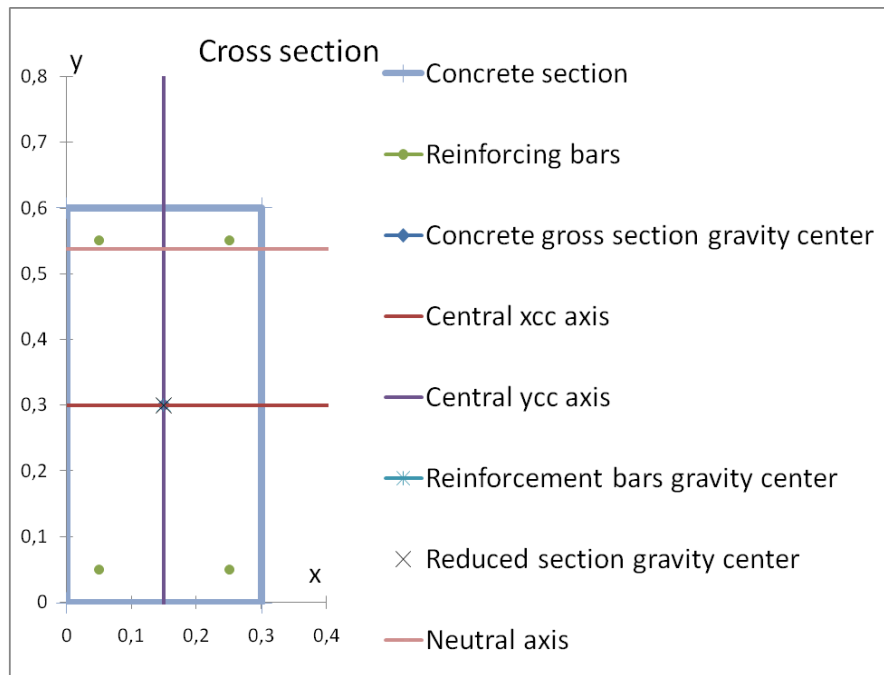
5.4.2 Search for:

- Results for reinforcement bars: N_s , M_{xs} , M_{ys}
- Results for concrete section: A_c , x_{cg} , y_{cg} , N_c , M_{xc} , M_{yc}
- Results for reduced section: N , M_x , M_y

5.4.3 Results from Component

- Results for reinforcement bars:
 $N_s = -392.40e3$
 $M_{xs} = -223.60e3$
 $M_{ys} = 0.00e3$
- Results for concrete section:
 $A_c = 0.0192$
 $x_{cg} = 0.00000000$
 $y_{cg} = 0.27717785$
 $N_c = 392.40e3$
 $M_{xc} = -108.76e3$
 $M_{yc} = 0.00e3$
- Results for reduced section:
 $N = 0.00e3$
 $M_x = -332.36e3$
 $M_y = 0.00e3$

5.4.4 Results (manual calculations):



- Results for reinforcement bars:
Distance between maximum compression corner of the section and neutral axis:
height of a section: $h = 0.6$
 $y_b = -Eps_top \cdot h / (Eps_bot - Eps_top) = 0.0639$
 $Eps_u1 = f_{yd} / E_s$

Concrete gross section gravity center:

$$x_{Cc} = 0.15$$

$$y_{Cc} = 0.30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar i:

If $|\text{eps_i}| \leq f_{yd} / E_s$ then
 $\text{sigma_i} = \text{eps_i} / \text{Eps_u1} \cdot f_{yd}$
 else
 $\text{sigma_i} = f_{yd} + (|\text{eps_i}| - \text{Eps_u1}) / (\text{Eps_u2} - \text{Eps_u1}) \cdot (k - 1) \cdot f_{yd}$

i	x	y	ϕ	Distance d from neutral axis	strain eps = d / y_b • Eps_top	stresses sigma / 10e6	force in bar N = sigma • $\pi \cdot \phi^2 / 4$	Positions in central axes coordinate system x_{cc} y_{cc}		$M_{xs} = -N$ • y_{cc}	$M_{ys} = N$ • x_{cc}
1	0,05	0,05	0,032	-0,486	-0,02663	-400	-321699	-0,10	-0,25	-80425	32170
2	0,25	0,05	0,032	-0,486	-0,02663	-400	-321699	0,10	-0,25	-80425	-32170
3	0,25	0,55	0,032	0,014	0,00076	156	125500	0,10	0,25	-31375	12550
4	0,05	0,55	0,032	0,014	0,00076	156	125500	-0,10	0,25	-31375	-12550
						Sum of forces in bars:	-392398			-	0

$$N_s = -392.40e3$$

$$M_{xs} = -223.60e3$$

$$M_{ys} = 0.00e3$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

$$\text{if } \text{eps} \leq \text{Eps_c1} \quad N_{sc} = f_{cd} \cdot \text{eps} / \text{Eps_c1} \cdot \text{Area}$$

$$\text{if } \text{eps} > \text{Eps_c1} \quad N_{sc} = f_{cd} \cdot \text{Area}$$

i	x	y	ϕ	strain eps	Positions in central axes coordinate system x_{cc} y_{cc}		Area of concrete with compression stress cut from the section in place of bars	Force in concrete in place of bars N_{sc}	$M_{xsc} = -$ $N_{sc} \cdot y_{cc}$	$M_{ysc} =$ $N_{sc} \cdot x_{cc}$
1	0,05	0,05	0,032	-0,02663	-0,10	-0,25				
2	0,25	0,05	0,032	-0,02663	0,10	-0,25				
3	0,25	0,55	0,032	0,00076	0,10	0,25	0,000804248	9183	-2296	918
4	0,05	0,55	0,032	0,00076	-0,10	0,25	0,000804248	9183	-2296	-918
							Sum of forces:	18366	-4591	0

Reduction forces in concrete:

$$N_{sc} = 18.37e3$$

$$M_{xsc} = -4.59e3$$

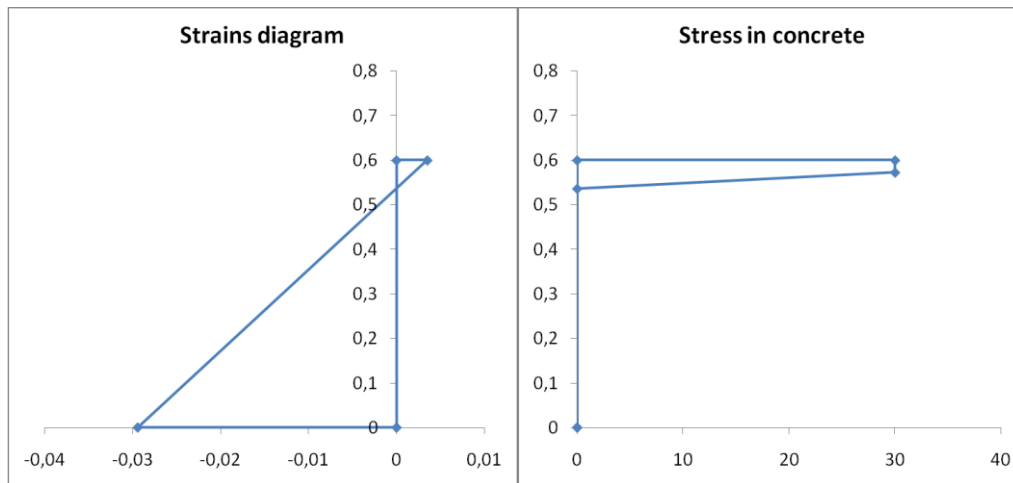
$$M_{ysc} = 0.00e3$$

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

$$x_{scg} = \text{sum}(x_{cc_i} \cdot \text{Area_i}) / \text{sum}(\text{Area_i}) = 0.00$$

$$y_{scg} = \text{sum}(y_{cc_i} \cdot \text{Area_i}) / \text{sum}(\text{Area_i}) = 0.25$$

- Results for concrete section:



Height of compressed part of the section:

$$y_b = 0.0639$$

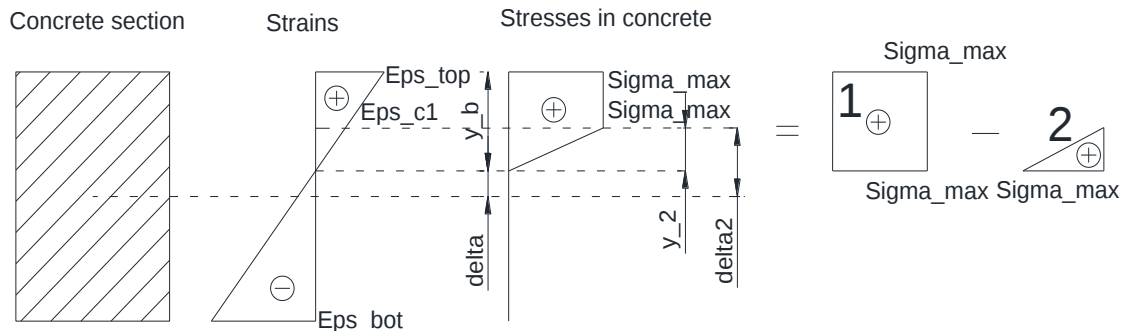
Area of contour under compressive stress:

$$A_c = 0.3 \cdot y_b = 0.0192$$

Axial force N_c in compressed part of concrete section is calculated by subtraction of volume of a triangular solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of N_c in Introduction).

$$N_{c'} = N_{c1} - N_{c2}$$

$$N_c = N_{c'} - N_{sc}$$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

Moments M_{xc} and M_{yc} caused by stresses acting on entire compressed gross area are calculated following below equations:

$$M_{xc} = - N_c \cdot y_{cg}$$

$$M_{yc} = N_c \cdot x_{cg}$$

$$\text{Sigma_max} = f_{cd} = 30.000e6$$

Distance between gravity center of contour and neutral axis:

$$\Delta = 0.6 / 2 - y_b = 0.2361$$

$$\text{Angle} = 3 / 2 \cdot \pi = 4.712$$

Distance between the gravity center of contour of the section and:

- the axis with strains equal Eps_c1 if Eps_top $\geq$ Eps_c1 or
- the axis parallel to neutral axis intersecting the most compressed corner of the section if Eps_top < Eps_c1,:

$$\text{delta2} = \text{delta} + \text{MIN} (y_b; \text{Eps_c1} / \text{Eps_top} \cdot y_b) = 0.2726$$

$$\text{Angle} = 3 / 2 \cdot \pi = 4.712$$

Height of the part of the section where concrete stress is not constant:

$$y_2 = \text{delta2} - \text{delta} = 0.0365$$

Axial force in compressed part of concrete gross section

$$N_{c1} = A_c \cdot \text{Sigma_max} = 0.5751\text{e6}$$

$$N_{c2} = 0.3 \cdot y_2 \cdot \text{Sigma_max} / 2 = 0.1643\text{e6}$$

$$N_c' = N_{c1} - N_{c2} = 0.41076\text{e6}$$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$$x_{c1} = 0$$

$$y_{c1} = \text{delta} + y_b / 2 = 0.26805$$

$$x_{c2} = 0$$

$$y_{c2} = \text{delta} + y_2 / 3 = 0.24827$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.000000$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.277178$$

Internal forces in concrete:

$$N_c = N_{c1} - N_{sc} = 392.40\text{e3}$$

$$M_{xc} = - N_c \cdot y_{cg} = - 108.76\text{e3}$$

$$M_{yc} = N_{c1} \cdot x_{cg} = 0.00\text{e3}$$

- Results for reduced section:

$$N = N_c + N_s = 0.00\text{e3}$$

$$M_x = M_{xc} + M_{xs} = -332.36\text{e3}$$

$$M_y = M_{yc} + M_{ys} = 0.00\text{e3}$$

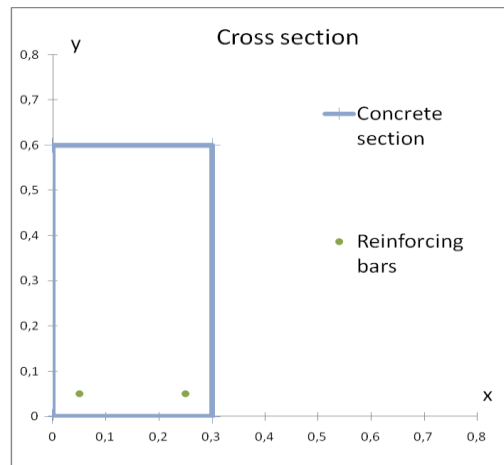
5.5 Case 4c

5.5.1 Data:

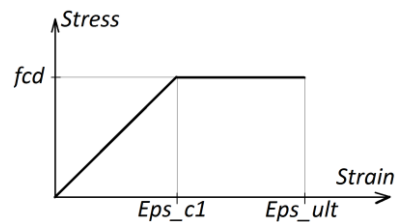
- Geometry:
 - Concrete section:

x = 0.0	y = 0.0
x = 0.0	y = 0.6
x = 0.3	y = 0.6
x = 0.3	y = 0.0
 - Reinforcing bars:

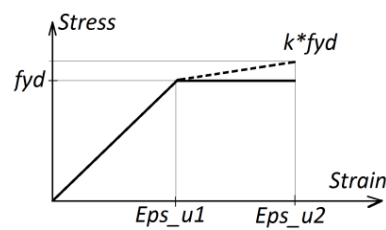
x = 0.05	y = 0.05	$\phi = 0.032$
x = 0.25	y = 0.05	$\phi = 0.032$



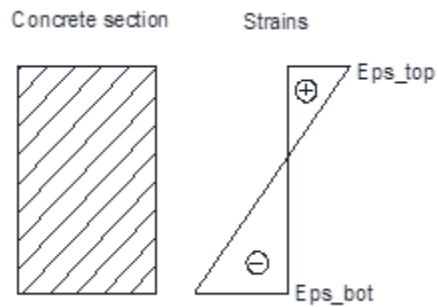
- Concrete parameters:
 - Design strength $f_{cd} = 30e6$
 - Modulus of elasticity $E_c = 32e9$
 - Strain-stress model: bilinear
 - Strain ultimate limit $Eps_ult = 0.0035$
 - Strain relation change over $Eps_c1 = 0.0020$



- Steel parameters:
 - Design strength $f_{yd} = 400e6$
 - Hardening factor $k = 1.0$
 - Modulus of elasticity $E_s = 205e9$
 - Strain ultimate limit $Eps_u2 = 0.1$



- Strains:
 - Top strain $Eps_top = 0.0015$
 - Bottom strain $Eps_bot = -0.002$
 - Neutral axis angle $Angle = \pi \cdot 3 / 2$



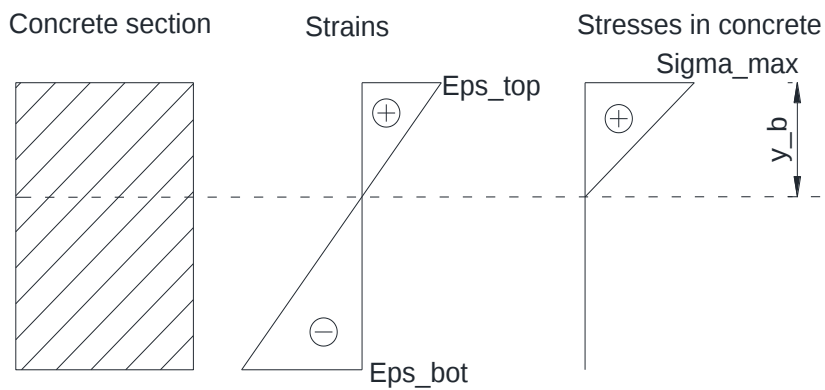
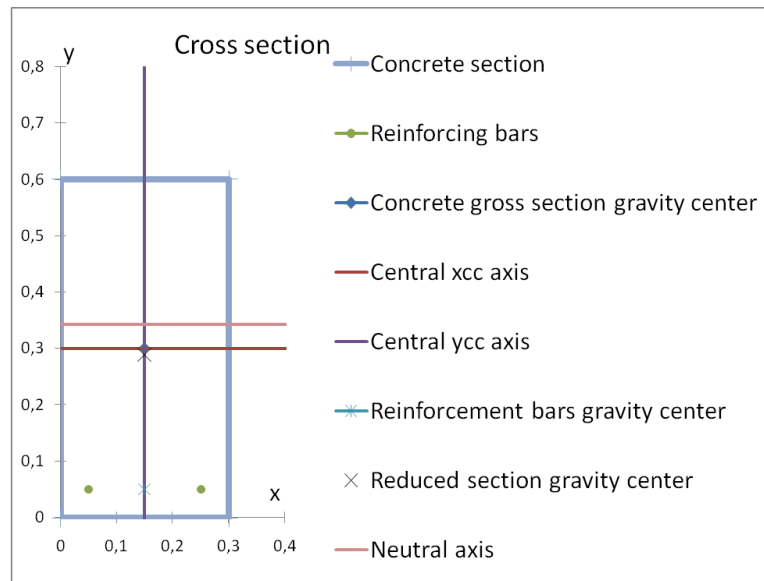
5.5.2 Search for:

- Results for reinforcement bars: N_s , M_{xs} , M_{ys}
- Results for concrete section: A_c , x_{cg} , y_{cg} , N_c , M_{xc} , M_{yc}
- Results for reduced section: N , M_x , M_y

5.5.3 Results from Component

- Results for reinforcement bars:
 $N_s = -563.31e3$
 $M_{xs} = -140.83e3$
 $M_{ys} = 0.00e3$
- Results for concrete section:
 $A_c = 0.0771$
 $x_{cg} = 0.000000$
 $y_{cg} = 0.21428571$
 $N_c = 867.86e3$
 $M_{xc} = -185.97e3$
 $M_{yc} = 0.00e3$
- Results for reduced section:
 $N = 304.55e3$
 $M_x = -326.80e3$
 $M_y = 0.00e3$

5.5.4 Results (manual calculations):



- Results for reinforcement bars:
 Distance between maximum compression corner of the section and neutral axis:
 height of a section: $h = 0.6$
 $y_b = -Eps_top \cdot h / (Eps_bot - Eps_top) = 0.257143$
 $Eps_u1 = f_{yd} / E_s$

Concrete gross section gravity center:

$$x_{Cc} = 0.15$$

$$y_{Cc} = 0.30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar i:

$$\text{If } |eps_i| \leq f_{yd} / E_s \text{ then} \\ \sigma_i = eps_i / Eps_u1 \cdot f_{yd} \\ \text{else}$$

$$\sigma_i = f_{yd} + (|\epsilon_i| - \epsilon_{u1}) / (\epsilon_{u2} - \epsilon_{u1}) \cdot (k - 1) \cdot f_{yd}$$

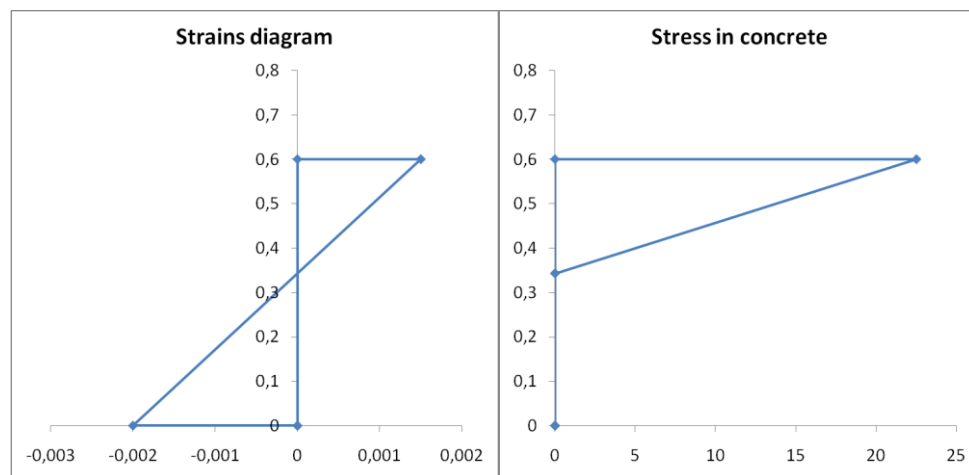
i	x	y	ϕ	Distance d from neutral axis	strain $\epsilon = d /$ $y_b \cdot$ ϵ_{top}	stresses $\sigma /$ 10e6	force in bar $N =$ $\sigma \cdot$ $\pi \cdot \phi^2 / 4$	Positions in central axes coordinate system x_{cc} y_{cc}		$M_{xs} = -N$ $\cdot y_{cc}$	$M_{ys} = N$ $\cdot x_{cc}$
1	0,05	0,05	0,032	-0,293	-0,00171	-350	-281654	-0,10	-0,25	-70414	28165
2	0,25	0,05	0,032	-0,293	-0,00171	-350	-281654	0,10	-0,25	-70414	-28165
Sum of forces in bars:							-563309			140827	0

$$N_s = -563.31e3$$

$$M_{xs} = -140.83e3$$

$$M_{ys} = 0.00e3$$

- Results for concrete section:



Height of compressed part of the section:

$$y_b = 0.257143$$

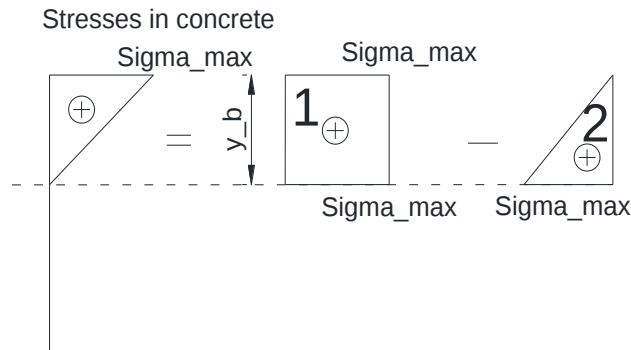
Area of contour under compressive stress:

$$A_c = 0.3 \cdot y_b = 0.0771$$

Axial force N_c in compressed part of concrete section is calculated by subtraction of volume of a triangular solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of N_c in Introduction).

$$N_c' = N_{c1} - N_{c2}$$

$$N_c = N_c'$$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2})$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2})$$

Moments M_{xc} and M_{yc} caused by stresses acting on entire compressed gross area are calculated following below equations:

$$M_{xc} = - N_c \cdot y_{cg}$$

$$M_{yc} = N_c \cdot x_{cg}$$

$$\text{Sigma_max} = f_{cd} \cdot \text{Eps_top} / \text{Eps_c1} = 22.500\text{e6}$$

Axial force in compressed part of concrete gross section

$$N_{c1} = A_c \cdot \text{Sigma_max} = 1.7357\text{e6}$$

$$N_{c2} = A_c \cdot \text{Sigma_max} / 2 = 0.8679\text{e6}$$

$$N_{c'} = N_{c1} - N_{c2} = 0.8679\text{e6}$$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$$x_{c1} = 0$$

$$y_{c1} = 0.3 - y_b / 2 = 0.17142857$$

$$x_{c2} = 0$$

$$y_{c2} = 0.3 - y_b \cdot 2/3 = 0.12857143$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2}) = 0.00000000$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2}) = 0.21428571$$

Internal forces in concrete:

$$N_c = N_{c'} = 867.86\text{e3}$$

$$M_{xc} = - N_c \cdot y_{cg} = -185.97\text{e3}$$

$$M_{yc} = N_{c'} \cdot x_{cg} = 0.00\text{e3}$$

- Results for reduced section:

$$N = N_c + N_s = 304.55\text{e3}$$

$$M_x = M_{xc} + M_{xs} = -326.80\text{e3}$$

$$M_y = M_{yc} + M_{ys} = 0.00\text{e3}$$

6 Case 5: Calculation of internal forces for given state of strain in the section and parabolic-rectangular model of concrete

6.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, extreme strains in the section and give internal forces.

6.2 Summary:

This sample gives as results the internal forces for:

- A rectangular concrete shape section with symmetric reinforcement (Case 5a and Case 5b)
- A rectangular concrete shape section with asymmetric reinforcement (Case 5c)
- A parabolic-rectangular model of concrete and horizontal branch model of steel
- A section under unidirectional bending with entire section under compression (Case 5a)
- A section under unidirectional bending with a part of section under compression (Case 5b and Case 5c)
- A state of strain with ultimate compressive strain in concrete (Case 5a and Case 5b)
- A strain below the ultimate compressive strain in concrete (Case 5c).

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel.

Based on input data, Component gives as output results:

- Results for reinforcement bars:
 - Axial force (N_s)
 - Moment with respect to x_{cc} axis (M_{xs})
 - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
 - Area of contour under compressive stress (A_c)
 - Center of gravity of stresses in the concrete (x_{cg} , y_{cg})
 - Axial force (N_c)
 - Moment with respect to x_{cc} axis (M_{xc})
 - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
 - Axial force (N)
 - Moment with respect to x_{cc} axis (M_x)
 - Moment with respect to y_{cc} axis (M_y)

For these results, x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

6.3 Case 5a

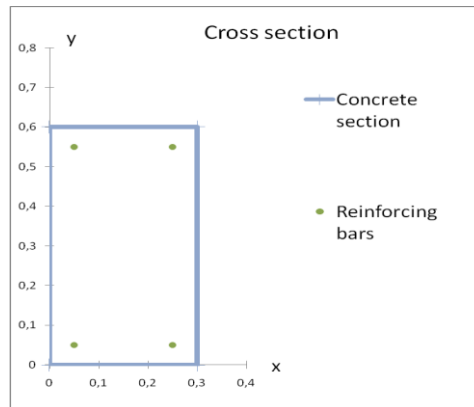
6.3.1 Data:

- Geometry:
 - Concrete section:

$x = 0.0$ $y = 0.0$
 $x = 0.0$ $y = 0.6$
 $x = 0.3$ $y = 0.6$
 $x = 0.3$ $y = 0.0$

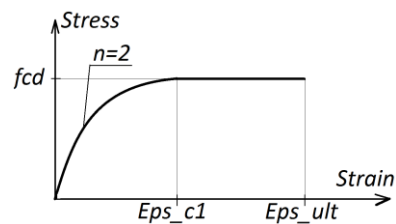
- Reinforcing bars:

$x = 0.05$ $y = 0.05$ $\phi = 0.016$
 $x = 0.05$ $y = 0.55$ $\phi = 0.016$
 $x = 0.25$ $y = 0.55$ $\phi = 0.016$
 $x = 0.25$ $y = 0.05$ $\phi = 0.016$



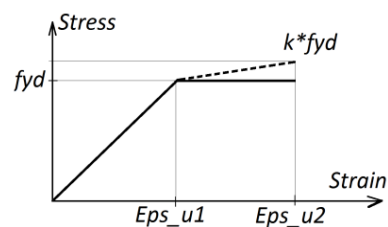
- Concrete parameters:

- Design strength $f_{cd} = 30e6$
- Modulus of elasticity $E_c = 32e9$
- Strain-stress model: parabolic-rectangular
- Strain ultimate limit $Eps_ult = 0.0035$
- Strain relation change over $Eps_c1 = 0.0020$

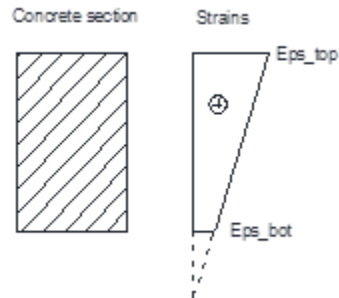


- Steel parameters:

- Design strength $f_{yd} = 400e6$
- Hardening factor $k = 1.0$
- Modulus of elasticity $E_s = 200e9$
- Strain ultimate limit $Eps_u2 = 0.1$



- Strains:
 - Top strain $\text{Eps_top} = 0.0035$
 - Bottom strain $\text{Eps_bot} = 0.0005$
 - Neutral axis angle $\text{Angle} = \pi \cdot 3 / 2$



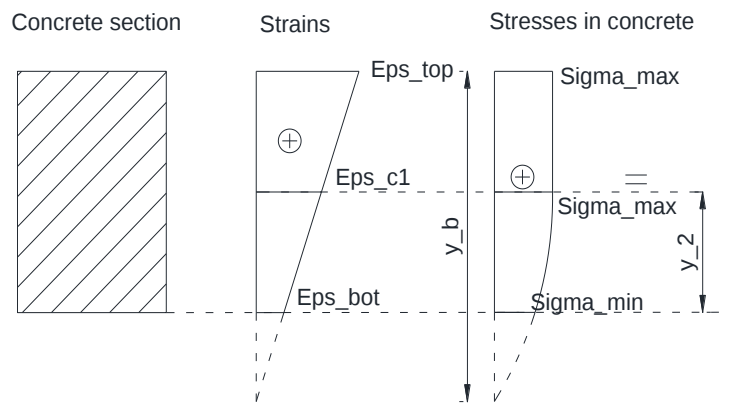
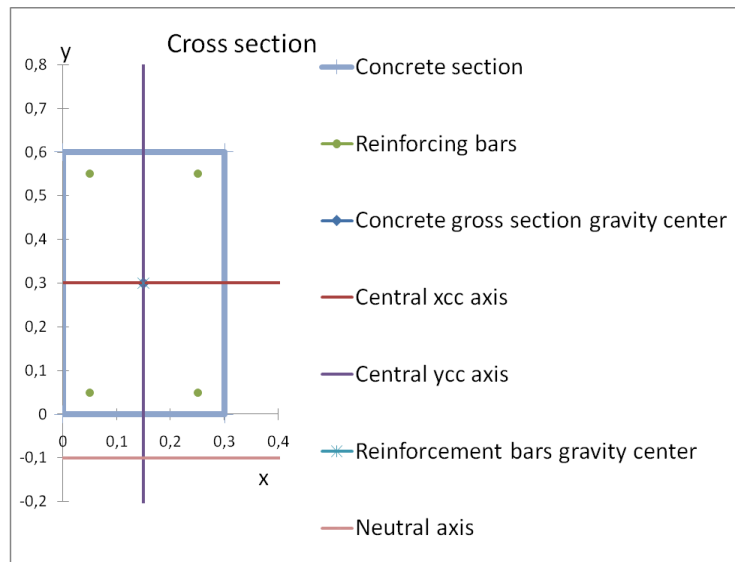
6.3.2 Search for:

- Results for reinforcement bars: N_s , M_{xs} , M_{ys}
- Results for concrete section: A_c , x_{cg} , y_{cg} , N_c , M_{xc} , M_{yc}
- Results for reduced section: N , M_x , M_y

6.3.3 Results from Component

- Results for reinforcement bars:
 - $N_s = 221.17e3$
 - $M_{xs} = -25.13e3$
 - $M_{ys} = 0.00e3$
- Results for concrete section:
 - $A_c = 0.1800$
 - $x_{cg} = 0.00000000$
 - $y_{cg} = 0.02312688$
 - $N_c = 4874.33e3$
 - $M_{xc} = -112.73e3$
 - $M_{yc} = 0.00e3$
- Results for reduced section:
 - $N = 5095.50e3$
 - $M_x = -137.86e3$
 - $M_y = 0.00e3$

6.3.4 Results (manual calculations):



- Results for reinforcement bars:
 Distance between maximum compression corner of the section and neutral axis:
 height of a section: $h = 0.6$
 $y_b = -Eps_top \cdot h / (Eps_bot - Eps_top) = 0.7$
 $Eps_u1 = f_{yd} / E_s$

Concrete gross section gravity center:

$$x_{Cc} = 0.15$$

$$y_{Cc} = 0.30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar i:

If $|eps_i| \leq f_{yd} / E_s$ then

$$sigma_i = eps_i / Eps_u1 \cdot f_{yd}$$

else

$$sigma_i = f_{yd} + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) \cdot (k - 1) \cdot f_{yd}$$

i	x	y	ϕ	Distance d from neutral axis	strain $\epsilon_s = d /$ $y_b \cdot$ Eps_top	stresses sigma / 10e6	force in bar $N =$ $\sigma \cdot \pi$ $\cdot \phi^2 / 4$	Positions in central axes coordinate system x_{cc} y_{cc}		$M_{xs} = -$ $N \cdot v$	$M_{ys} =$ $N \cdot x_{cc}$
1	0,05	0,05	0,016	0,150	0,00075	150	30159	-0,10	-0,25	7540	-3016
2	0,25	0,05	0,016	0,150	0,00075	150	30159	0,10	-0,25	7540	3016
3	0,25	0,55	0,016	0,650	0,00325	400	80425	0,10	0,25	-20106	8042
4	0,05	0,55	0,016	0,650	0,00325	400	80425	-0,10	0,25	-20106	-8042
						Sum of forces in bars:	221168			-25133	0

$$N_s = 221.17e3$$

$$M_{xs} = -25.13e3$$

$$M_{ys} = 0.00e3$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

$$\text{if } \epsilon_s \leq \epsilon_{sc1} \text{ then } N_{sc} = f_{cd} \cdot (1 - (1 - \epsilon_s / \epsilon_{sc1})^2) \cdot \text{Area}$$

$$\text{if } \epsilon_s > \epsilon_{sc1} \text{ then } N_{sc} = f_{cd} \cdot \text{Area}$$

i	x	y	ϕ	strain eps	Positions in central axes coordinate system x_{cc} y_{cc}		Area of concrete with compression stress cut from the section in place of bars	Force in concrete in place of bars N_{sc}	$M_{xsc} = -$ $N_{sc} \cdot y_{cc}$	$M_{ysc} =$ $N_{sc} \cdot x_{cc}$
1	0,05	0,05	0,016	0,00075	-0,1	-0,25	0,000201062	3676	919	-226
2	0,25	0,05	0,016	0,00075	0,1	-0,25	0,000201062	3676	919	226
3	0,25	0,55	0,016	0,00325	0,1	0,25	0,000201062	6032	-1508	603
4	0,05	0,55	0,016	0,00325	-0,1	0,25	0,000201062	6032	-1508	-603
							Sum of forces:	19415	-1178	0

Reduction forces in concrete:

$$N_{sc} = 19.42e3$$

$$M_{xsc} = -1.18e3$$

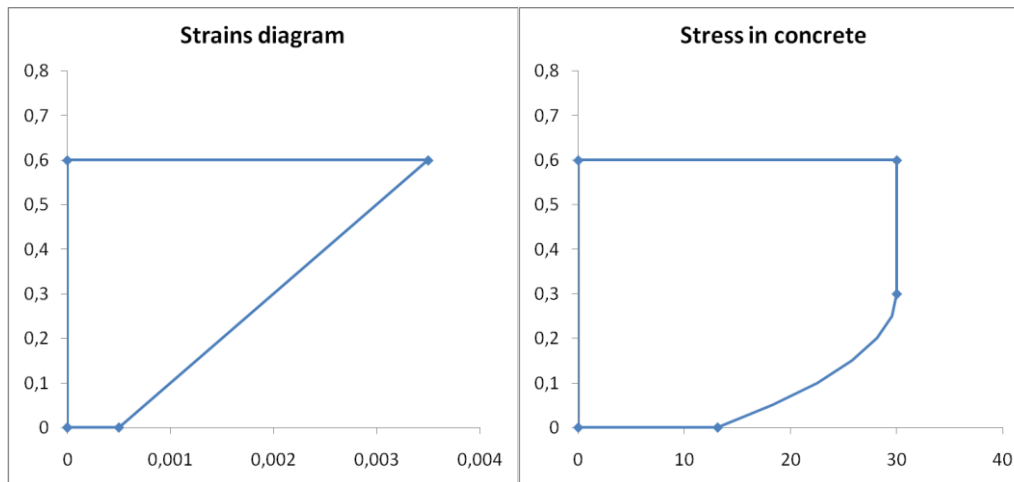
$$M_{ysc} = 0.00e3$$

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

$$x_{scg} = \sum(x_{cc_i} \cdot \text{Area_i}) / \sum(\text{Area_i}) = 0.00000$$

$$y_{scg} = \sum(y_{cc_i} \cdot \text{Area_i}) / \sum(\text{Area_i}) = 0.06068$$

- Results for concrete section:



Height of compressed part of the section:

$$0.6000$$

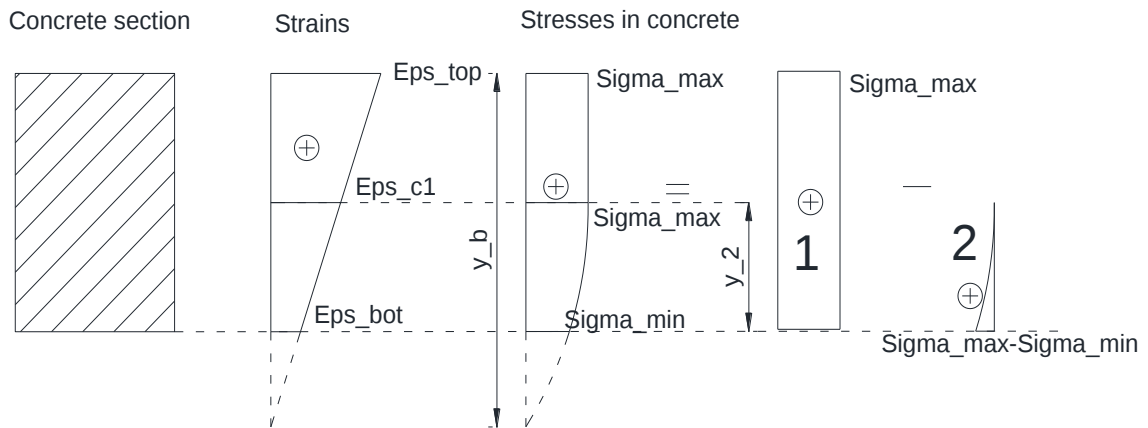
Area of contour under compressive stress:

$$A_c = 0.3 \cdot 0.6000 = 0.1800$$

Axial force N_c in compressed part of concrete section is calculated by subtraction of volume of a parabolic solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of N_c in Introduction)

$$N_{c'} = N_{c1} - N_{c2}$$

$$N_c = N_{c'} - N_{sc}$$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

Moments M_{xc} and M_{yc} caused by stresses acting on entire compressed gross area are calculated following below equations:

$$M_{xc} = - N_c \cdot y_{cg}$$

$$M_{yc} = N_c \cdot x_{cg}$$

$$\text{Sigma_max} = f_{cd} = 30.000e6$$

$$\text{Sigma_min} = f_{cd} \cdot (1 - (1 - \text{Eps_bot} / \text{Eps_c1})^2) = 13.125\text{e6}$$

Distance between gravity center of contour and neutral axis:

$$\text{Delta} = - (0.6 / 2 - y_b) = 0.4$$

$$\text{Angle} = 3 / 2 \cdot \pi = 4.712$$

Distance between the gravity center of contour of the section and:

- the axis with strains equal Eps_c1 if $\text{Eps_top} \geq \text{Eps_c1}$ or

- the axis parallel to neutral axis intersecting the most compressed corner of the section if $\text{Eps_top} < \text{Eps_c1}$,:

$$\text{delta2} = - \text{delta} + \text{MIN}(y_b; \text{Eps_c1} / \text{Eps_top} \cdot y_b) = 0$$

$$\text{Angle} = 3 / 2 \cdot \pi = 4.712$$

Height of the part of the section where concrete stress is not constant:

$$y_2 = 0.3 - \text{delta2} = 0.3$$

Axial force in compressed part of concrete gross section

$$N_{c1} = A_c \cdot \text{Sigma_max} = 5.400\text{e6}$$

$$N_{c2} = 0.3 \cdot 1 / 3 \cdot y_2 \cdot (\text{Sigma_max} - \text{Sigma_min}) = 0.50625\text{e6}$$

$$N_c = N_{c1} - N_{c2} = 4.89375\text{e6}$$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$$x_{c1} = 0$$

$$y_{c1} = 0$$

$$x_{c2} = 0$$

$$y_{c2} = - 0.3 + y_2 / 4 = - 0.225$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.000000$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.023127$$

Internal forces in concrete:

$$N_c = N_{c'} - N_{sc} = 4874.33\text{e3}$$

$$M_{xc} = - N_c \cdot y_{cg} = -112.73\text{e3}$$

$$M_{yc} = N_c \cdot x_{cg} = 0.00\text{e3}$$

- Results for reduced section:

$$N = N_c + N_s = 5095.50\text{e3}$$

$$M_x = M_{xc} + M_{xs} = -137.86\text{e3}$$

$$M_y = M_{yc} + M_{ys} = 0.00\text{e3}$$

6.4 Case 5b

6.4.1 Data:

- Geometry:

- Concrete section:

$$x = 0.0 \quad y = 0.0$$

$$x = 0.0 \quad y = 0.6$$

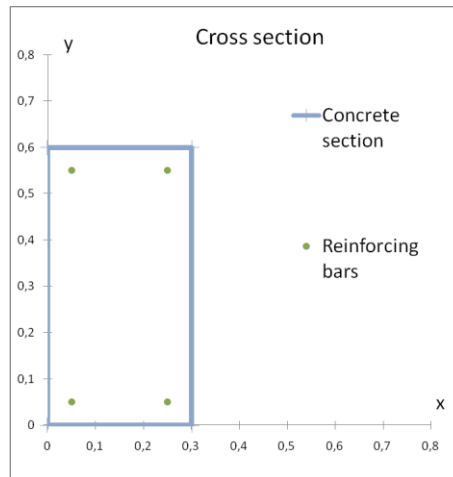
$$x = 0.3 \quad y = 0.6$$

$$x = 0.3 \quad y = 0.0$$

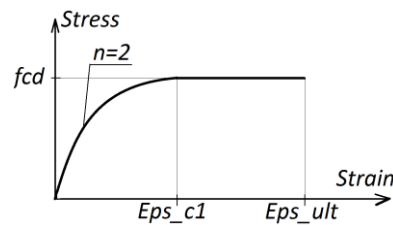
- Reinforcing bars:

$$x = 0.05 \quad y = 0.05 \quad \phi = 0.032$$

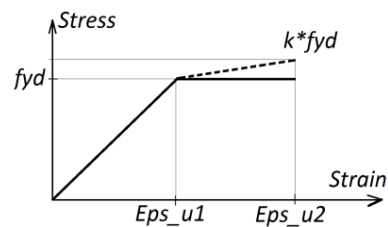
$x = 0.05$	$y = 0.55$	$\phi = 0.032$
$x = 0.25$	$y = 0.55$	$\phi = 0.032$
$x = 0.25$	$y = 0.05$	$\phi = 0.032$



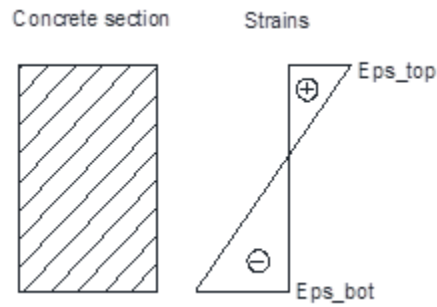
- Concrete parameters:
 - Design strength $f_{cd} = 30e6$
 - Modulus of elasticity $E_c = 32e9$
 - Strain-stress model: parabolic-rectangular
 - Strain ultimate limit $Eps_ult = 0.0035$
 - Strain relation change over $Eps_c1 = 0.0020$



- Steel parameters:
 - Design strength $f_{yd} = 400e6$
 - Hardening factor $k = 1.0$
 - Modulus of elasticity $E_s = 200e9$
 - Strain ultimate limit $Eps_u2 = 0.1$



- Strains:
 - Top strain $Eps_top = 0.0035$
 - Bottom strain $Eps_bot = -0.03034666$
 - Neutral axis angle $Angle = \pi \cdot 3 / 2$



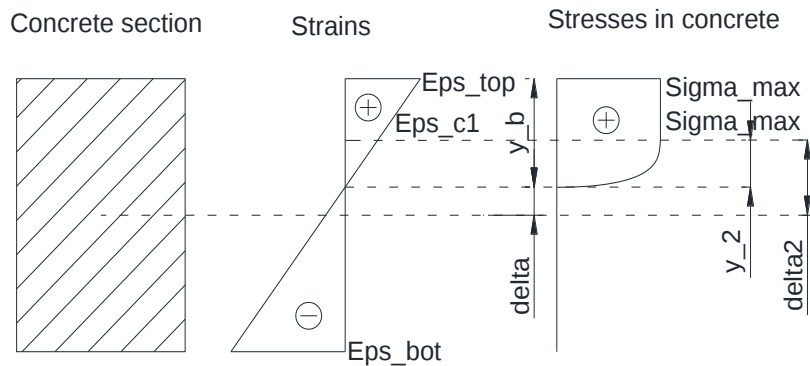
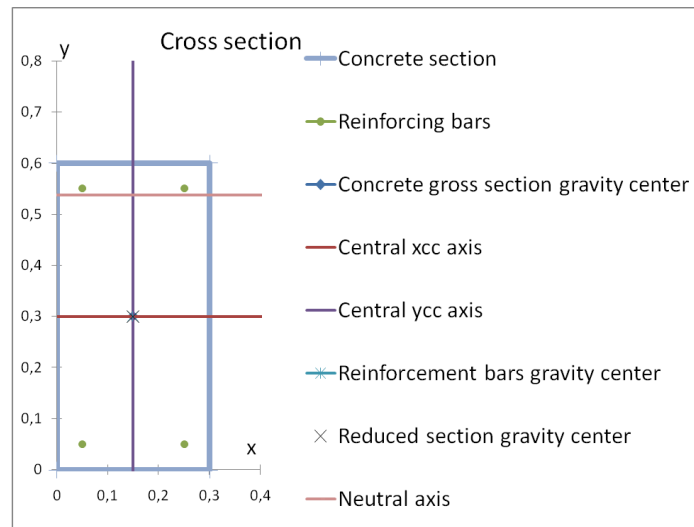
6.4.2 Search for:

- Results for reinforcement bars: N_s , M_{xs} , M_{ys}
- Results for concrete section: A_c , x_{cg} , y_{cg} , N_c , M_{xc} , M_{yc}
- Results for reduced section: N , M_x , M_y

6.4.3 Results from Component

- Results for reinforcement bars:
 $N_s = -424.82e3$
 $M_{xs} = -215.49e3$
 $M_{ys} = 0.00e3$
- Results for concrete section:
 $A_c = 0.0186$
 $x_{cg} = 0.00000000$
 $y_{cg} = 0.27574146$
 $N_c = 424.82e3$
 $M_{xc} = -117.14e3$
 $M_{yc} = 0.00e3$
- Results for reduced section:
 $N = 0.00e3$
 $M_x = -332.63e3$
 $M_y = 0.00e3$

6.4.4 Results (manual calculations):



- Results for reinforcement bars:
Distance between maximum compression corner of the section and neutral axis:
height of a section: $h = 0.6$
 $y_b = -Eps_top \cdot h / (Eps_bot - Eps_top) = 0.0620$
 $Eps_u1 = f_{yd} / E_s$

Concrete gross section gravity center:

$$x_{Cc} = 0.15$$

$$y_{Cc} = 0.30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar i:

If $|eps_i| \leq f_{yd} / E_s$ then

$$sigma_i = eps_i / Eps_u1 \cdot f_{yd}$$

else

$$sigma_i = f_{yd} + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) \cdot (k - 1) \cdot f_{yd}$$

i	x	y	ϕ	Distance d from neutral axis	Strain $\epsilon_s = d / y_b \cdot \epsilon_{s_top}$	Stresses $\sigma_s / 10e6$	Force in bar $N = \sigma_s \cdot \pi \cdot \phi^2 / 4$	Positions in central axes coordinate system x_{cc} y_{cc}		$M_{xs} = -N \cdot y_{cc}$	$M_{ys} = N \cdot x_{cc}$
1	0,05	0,05	0,032	-0,488	-0,02753	-400	-321699	-0,10	-0,25	-80425	32170
2	0,25	0,05	0,032	-0,488	-0,02753	-400	-321699	0,10	-0,25	-80425	-32170
3	0,25	0,55	0,032	0,012	0,00068	136	109288	0,10	0,25	-27322	10929
4	0,05	0,55	0,032	0,012	0,00068	136	109288	-0,10	0,25	-27322	-109290
						Sum of forces in bars:	-424821			215494	0

$$N_s = -424.82e3$$

$$M_{xs} = -215.49e3$$

$$M_{ys} = 0.00e3$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

$$\text{if } \epsilon_s \leq \epsilon_{sc1} \text{ then } N_{sc} = f_{cd} \cdot (1 - (1 - \epsilon_s / \epsilon_{sc1})^2) \cdot \text{Area}$$

$$\text{if } \epsilon_s > \epsilon_{sc1} \text{ then } N_{sc} = f_{cd} \cdot \text{Area}$$

i	x	y	ϕ	Strain ϵ_s	Positions in central axes coordinate system x_{cc} y_{cc}		Area of concrete with compression stress cut from the section in place of bars	Force in concrete in place of bars N_{sc}	$M_{xsc} = -N_{sc} \cdot y_{cc}$	$M_{ysc} = N_{sc} \cdot x_{cc}$
1	0,05	0,05	0,032	-0,02753	-0,10	-0,25				
2	0,25	0,05	0,032	-0,02753	0,10	-0,25				
3	0,25	0,55	0,032	0,00068	0,10	0,25	0,000804248	13609	-3402	1361
4	0,05	0,55	0,032	0,00068	-0,10	0,25	0,000804248	13609	-3402	-1361
							Sum of forces:	27217	-6804	0

Reduction forces in concrete:

$$N_{sc} = 27.22e3$$

$$M_{xsc} = -6.80e3$$

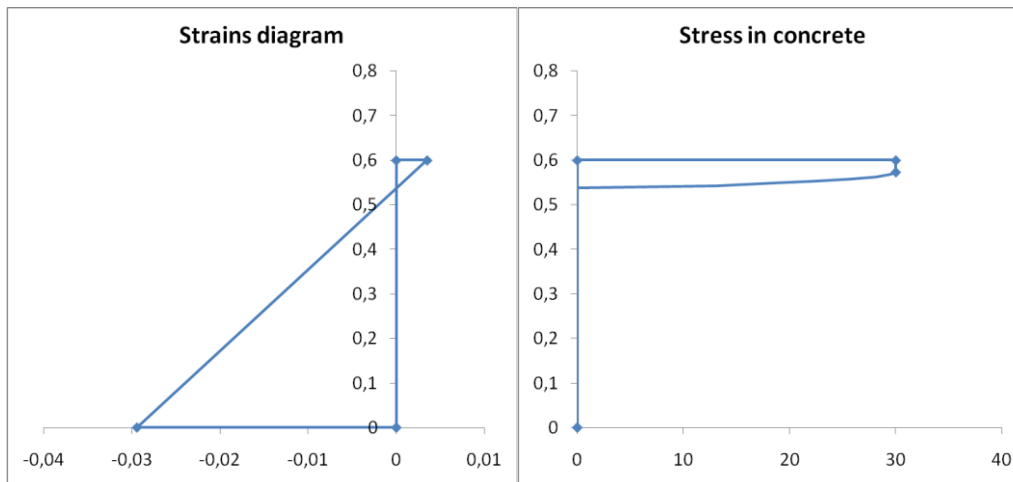
$$M_{ysc} = 0.00e3$$

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

$$x_{scg} = \sum(x_{cc_i} \cdot \text{Area_i}) / \sum(\text{Area_i}) = 0.00$$

$$y_{scg} = \sum(y_{cc_i} \cdot \text{Area_i}) / \sum(\text{Area_i}) = 0.25$$

- Results for concrete section:



Height of compressed part of the section:

$$y_b = 0.0620$$

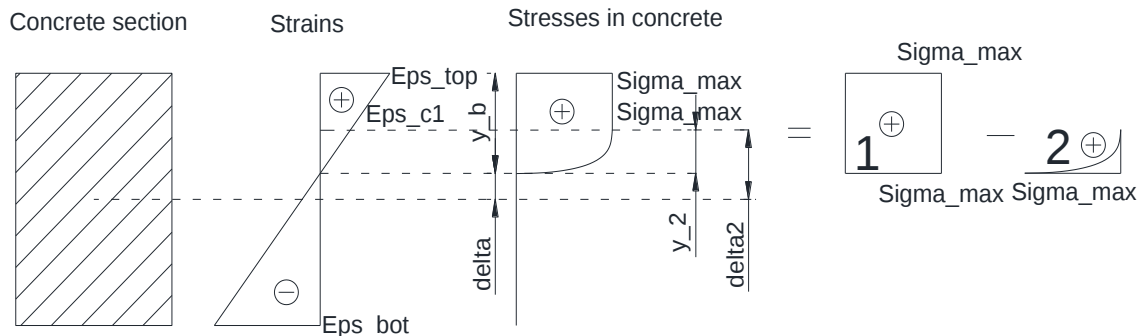
Area of contour under compressive stress:

$$A_c = 0.3 \cdot y_b = 0.0186$$

Axial force N_c in compressed part of concrete section is calculated by subtraction of volume of a parabolic solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of N_c in Introduction).

$$N_{c'} = N_{c1} - N_{c2}$$

$$N_c = N_{c'} - N_{sc}$$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

Moments M_{xc} and M_{yc} caused by stresses acting on entire compressed gross area are calculated following below equations:

$$M_{xc} = - N_c \cdot y_{cg}$$

$$M_{yc} = N_c \cdot x_{cg}$$

$$\text{Sigma_max} = f_{cd} = 30.000e6$$

Distance between gravity center of contour and neutral axis:

$$\text{delta} = 0.6 / 2 - y_b = 0.2380$$

$$\text{Angle} = 3 / 2 \cdot \pi = 4.712$$

Distance between the gravity center of contour of the section and:

- the axis with strains equal Eps_c1 if Eps_top $\geq$ Eps_c1 or
- the axis parallel to neutral axis intersecting the most compressed corner of the section if Eps_top < Eps_c1,:

$$\text{delta2} = \text{delta} + \text{MIN} (y_b; \text{Eps_c1} / \text{Eps_top} \cdot y_b) = 0.2734$$

$$\text{Angle} = 3 / 2 \cdot \pi = 4.712$$

Height of the part of the section where concrete stress is not constant:

$$y_2 = \text{delta2} - \text{delta} = 0.0354$$

Axial force in compressed part of concrete gross section

$$N_{c1} = A_c \cdot \text{Sigma_max} = 0.558401\text{e6}$$

$$N_{c2} = 0.3 \cdot y_2 \cdot \text{Sigma_max} / 3 = 0.106362\text{e6}$$

$$N_{c'} = N_{c1} - N_{c2} = 0.452039\text{e6}$$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$$x_{c1} = 0$$

$$y_{c1} = \text{delta} + y_b / 2 = 0.268978$$

$$x_{c2} = 0$$

$$y_{c2} = \text{delta} + y_2 / 3 = 0.246819$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.000000$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.275741$$

Internal forces in concrete:

$$N_c = N_{c'} - N_{sc} = 424.82\text{e3}$$

$$M_{xc} = - N_c \cdot y_{cg} = -117.14\text{e3}$$

$$M_{yc} = N_{c'} \cdot x_{cg} = 0.00\text{e3}$$

- Results for reduced section:

$$N = N_c + N_s = 0.00\text{e3}$$

$$M_x = M_{xc} + M_{xs} = -332.63\text{e3}$$

$$M_y = M_{yc} + M_{ys} = 0.00\text{e3}$$

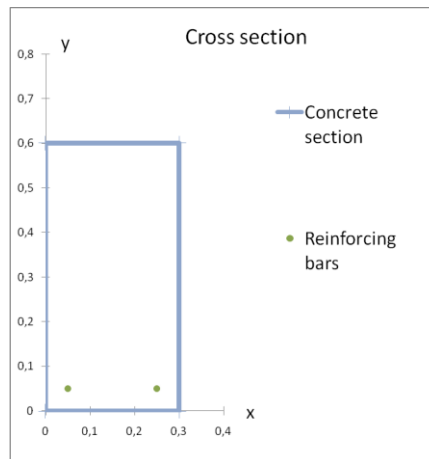
6.5 Case 5c

6.5.1 Data:

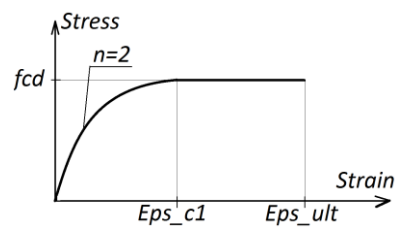
- Geometry:
 - Concrete section:

x = 0.0	y = 0.0
x = 0.0	y = 0.6
x = 0.3	y = 0.6
x = 0.3	y = 0.0
 - Reinforcing bars:

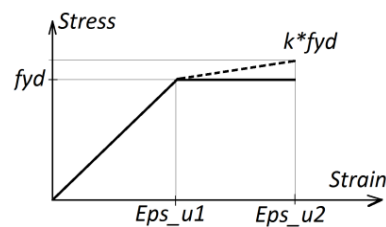
x = 0.05	y = 0.05	$\phi = 0.032$
x = 0.25	y = 0.05	$\phi = 0.032$



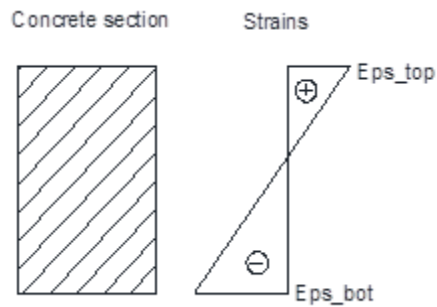
- Concrete parameters:
 - Design strength $f_{cd} = 30e6$
 - Modulus of elasticity $E_c = 32e9$
 - Strain-stress model: parabolic-rectangular
 - Strain ultimate limit $Eps_ult = 0.0035$
 - Strain relation change over $Eps_c1 = 0.0020$



- Steel parameters:
 - Design strength $f_{yd} = 400e6$
 - Hardening factor $k = 1.0$
 - Modulus of elasticity $E_s = 200e9$
 - Strain ultimate limit $Eps_u2 = 0.1$



- Strains:
 - Top strain $Eps_top = 0.0015$
 - Bottom strain $Eps_bot = -0.002$
 - Neutral axis angle $Angle = \pi \cdot 3 / 2$



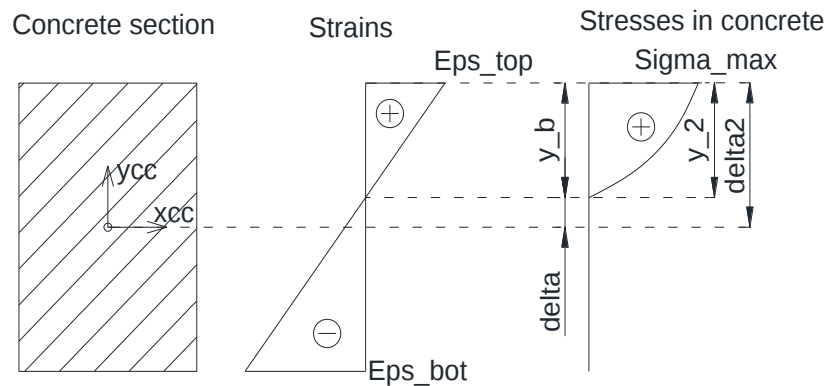
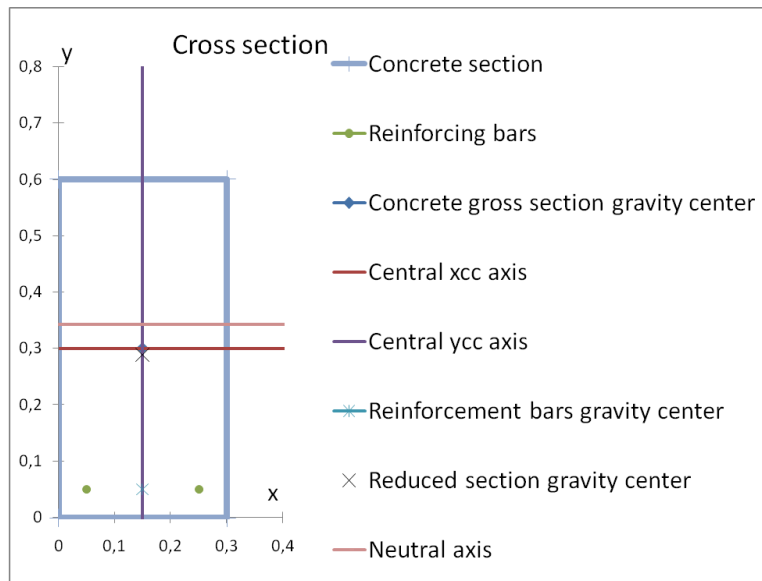
6.5.2 Search for:

- Results for reinforcement bars: N_s , M_{xs} , M_{ys}
- Results for concrete section: A_c , x_{cg} , y_{cg} , N_c , M_{xc} , M_{yc}
- Results for reduced section: N , M_x , M_y

6.5.3 Results from Component

- Results for reinforcement bars:
 $N_s = -549.57e3$
 $M_{xs} = -137.39e3$
 $M_{ys} = 0.00e3$
- Results for concrete section:
 $A_c = 0.0771$
 $x_{cg} = 0.000000$
 $y_{cg} = 0.20714286$
 $N_c = 1301.79e3$
 $M_{xc} = -269.66e3$
 $M_{yc} = 0.00e3$
- Results for reduced section:
 $N = 752.22e3$
 $M_x = -407.05e3$
 $M_y = 0.00e3$

6.5.4 Results (manual calculations):



- Results for reinforcement bars:
Distance between maximum compression corner of the section and neutral axis:
height of a section: $h = 0.6$
 $y_b = -Eps_top \cdot h / (Eps_bot - Eps_top) = 0.257143$
 $Eps_u1 = f_{yd} / E_s$

Concrete gross section gravity center:

$$x_{Cc} = 0.15$$

$$y_{Cc} = 0.30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar i:

If $|eps_i| \leq f_{yd} / E_s$ then

$$sigma_i = eps_i / Eps_u1 \cdot f_{yd}$$

else

$$sigma_i = f_{yd} + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) \cdot (k - 1) \cdot f_{yd}$$

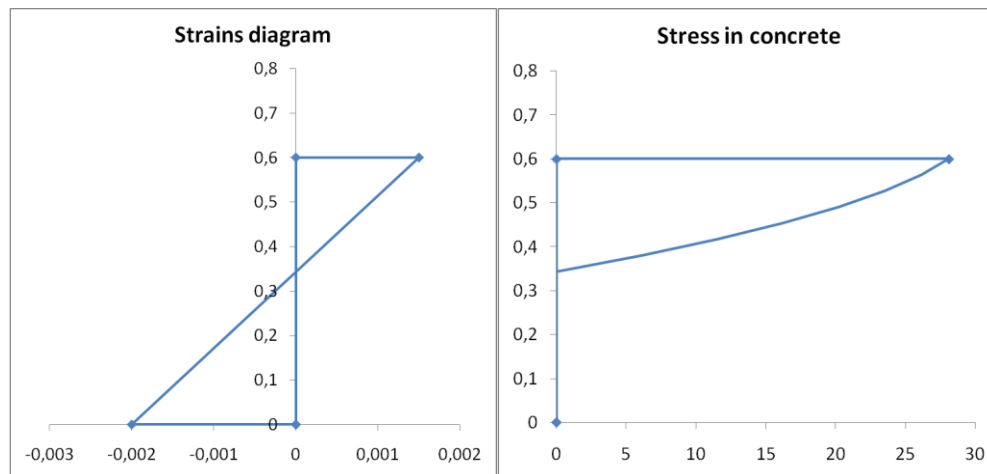
i	x	y	ϕ	Distance d from neutral axis	strain $\epsilon = d /$ $y_b \cdot$ E_{ps_top}	stresses $\sigma /$ 10e6	force in bar $N =$ $\sigma \cdot$ $\pi \cdot \phi^2 / 4$	Positions in central axes coordinate system x_{cc} y_{cc}		$M_{xs} = -N$ $\cdot y_{cc}$	$M_{ys} = N$ $\cdot x_{cc}$
1	0,05	0,05	0,032	-0,293	-0,00171	-342	-274785	-0,10	-0,25	-68696	27478
2	0,25	0,05	0,032	-0,293	-0,00171	-342	-274785	0,10	-0,25	-68696	-27478
Sum of forces in bars:							-549569			- 137392	0

$$N_s = -549.57e3$$

$$M_{xs} = -137.39e3$$

$$M_{ys} = 0.00e3$$

- Results for concrete section:



Height of compressed part of the section:

$$y_b = 0.257143$$

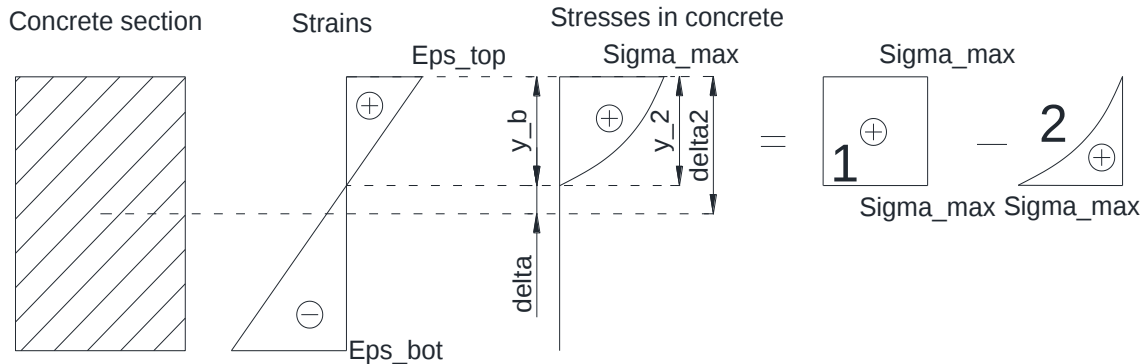
Area of contour under compressive stress:

$$A_c = 0.3 \cdot y_b = 0.0771$$

Axial force N_c in compressed part of concrete section is calculated by subtraction of volume of a parabolic solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of N_c in Introduction).

$$N_{c'} = N_{c1} - N_{c2}$$

$$N_c = N_{c'}$$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as weights.

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2})$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2})$$

Moments M_{xc} and M_{yc} caused by stresses acting on entire compressed gross area are calculated following below equations:

$$M_{xc} = - N_c \cdot y_{cg}$$

$$M_{yc} = N_c \cdot x_{cg}$$

$$\text{Sigma_max} = f_{cd} \cdot (1 - (1 - \text{Eps_top} / \text{Eps_c1})^2) = 28.125e6$$

Distance between gravity center of contour and neutral axis:

$$\text{delta} = 0.6 / 2 - y_b = 0.042857$$

$$\text{Angle} = 3 / 2 \cdot \pi = 4.712$$

Distance between the gravity center of contour of the section and:

- the axis with strains equal Eps_c1 if $\text{Eps_top} \geq \text{Eps_c1}$ or

- the axis parallel to neutral axis intersecting the most compressed corner of the section if

$\text{Eps_top} < \text{Eps_c1}$,:

$$\text{delta2} = \text{delta} + \text{MIN}(y_b; \text{Eps_c1} / \text{Eps_top} \cdot y_b) = 0.3000$$

$$\text{Angle} = 3 / 2 \cdot \pi = 4.712$$

Height of the part of the section where concrete stress is not constant:

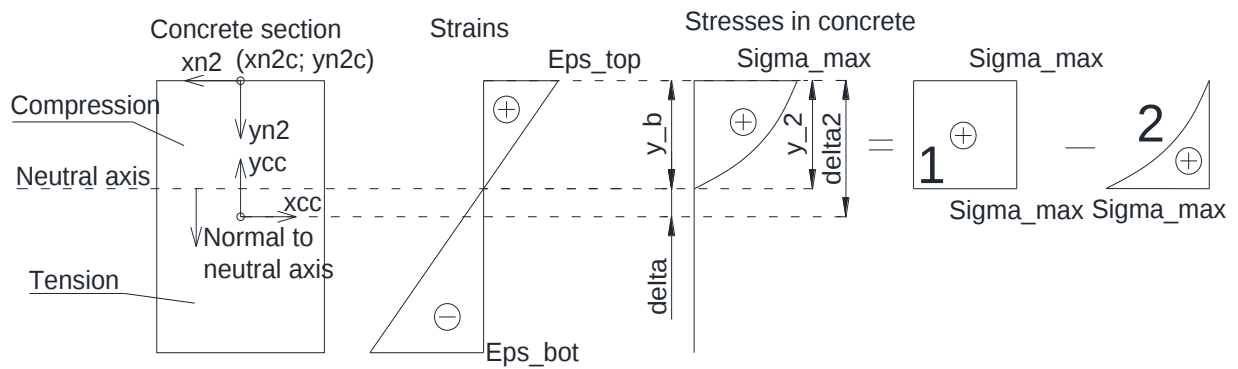
$$y_2 = \text{delta2} - \text{delta} = 0.257143$$

Axial force in compressed part of concrete gross section

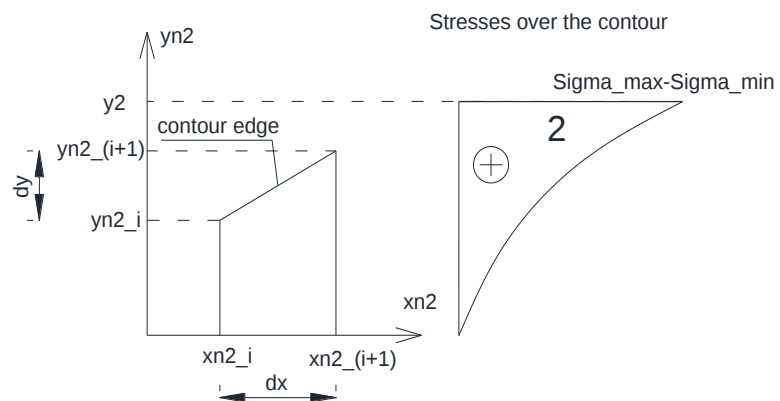
$$N_{c1} = A_c \cdot \text{Sigma_max} = 2.169643e6$$

To calculate N_{c2} we need to calculate an integral of stresses over contour for which stresses are changing according to the shape 2 on picture below. To do this we define a new coordinate system x_{n2} , y_{n2} (see picture below), where x_{n2} is an axis parallel to neutral axis, in distance delta2 from the gravity center of contour of the section, and it crosses axis y_{n2} in point $(x_{n2c}; y_{n2c})$ given in central axis coordinate system.

$$(x_{n2c}; y_{n2c}) = (0; \text{delta2})$$



Now we find coordinates of contour under stress solid 2 in (x_{n2}, y_{n2}) coordinates system and calculate an integral of stresses over this contour.



i	Coordinates in (x,y) system		Coordinates in central axes system		Coordinates in (x _{n2} , y _{n2}) system		dx	dy	A	B	N _{c2}	Static moment of stress solid MS _{yn2,zn2}	Static moment of stress solid MS _{xn2,zn2}
	x	y	x _{cc}	y _{cc}	x _{n2}	y _{n2}							
1	0,30	0,60	0,15	0,30	-0,15	0,00	0,00	0,26	0,00	0,00	0,00	0,00	0,00
2	0,30	0,34	0,15	0,04	-0,15	0,26	0,30	0,00	0,00	0,26	0,87	0,00	0,16
3	0,00	0,34	-0,15	0,04	0,15	0,26	0,00	-0,26	0,00	0,00	0,00	0,00	0,00
4	0,00	0,60	-0,15	0,30	0,15	0,00	0,30	0,00	0,00	0,00	0,00	0,00	0,00
N _{c2} = sum(N _{c2 i}) =											0,87		
C=										0,34			
D=										0,26			
gravity center of stress solid 2 in (x _{n2} ; y _{n2}) coordinates system												x _{c2'}	0,0000
												y _{c2'}	0,1821
gravity center of stress solid 2 in central axes (x _{cc} ; y _{cc}) coordinates system												x _{c2}	0,0000
												y _{c2}	0,1179

Where:

$$X_{cc} = X - X_{Cc}$$

$$Y_{cc} = Y - Y_{Cc}$$

$$X_{n2} = \cos(\text{Angle} - \pi / 2) \cdot (X_{cc} - X_{n2c}) + \sin(\text{Angle} - \pi / 2) \cdot (Y_{cc} - Y_{n2c})$$

$$Y_{n2} = -\sin(\text{Angle} - \pi / 2) \cdot (X_{cc} - X_{n2c}) + \cos(\text{Angle} - \pi / 2) \cdot (Y_{cc} - Y_{n2c})$$

$$dx = X_{i+1} - X_i$$

$$dy = Y_{i+1} - Y_i$$

$$A = dy/dx \text{ if } dx \neq 0 \text{ or } A=0 \text{ if } dx=0$$

$$B = -dy/dx \cdot X_{n2}(i+1) + Y_{n2}(i+1) \text{ if } dx \neq 0 \text{ or } B = 0 \text{ if } dx = 0$$

$$C = y_b \cdot \text{Eps}_{c1} / \text{Eps}_{top}$$

$$D = \text{MIN}(\text{Eps}_{c1}; \text{Eps}_{top}) \cdot y_b / \text{Eps}_{top}$$

$$n = 2 \text{ (power of stress-strain relation curve)}$$

$$d\text{Sigma} = \text{Sigma}_{max} - \text{Sigma}_{min}$$

$$N_{c2_i} = 0 \text{ if } A = 0 \text{ and } B = 0 \text{ or}$$

$$N_{c2_i} = f_{cd} \cdot C / (n+1) \cdot (C / (n+2) / A \cdot ((1 - (-Y_{n2}(i+1) + D) / C)^{n+2} - (1 - (-Y_{n2}(i) + D) / C)^{n+2}) - (X_{n2}(i+1) - X_{n2}(i)) \cdot (1 - D / C)^{n+1}) + (d\text{Sigma} - f_{cd}) \cdot (A / 2 \cdot (X_{n2}(i+1)^2 - X_{n2}(i)^2) + B \cdot (X_{n2}(i+1) - X_{n2}(i))) \text{ if } A \neq 0 \text{ or}$$

$$N_{c2_i} = (f_{cd} \cdot C / (n+1) \cdot ((1 - (-B + D) / C)^{n+1} - (1 - D / C)^{n+1})) \cdot dx + (-f_{cd} + d\text{Sigma}) \cdot B \cdot dx \text{ if } A = 0 \text{ and } B \neq 0$$

$$N_{c2} = \text{sum}(N_{c2_i})$$

$$MS_{yn2,zn2_i} = f_{cd} \cdot C / (n+1) \cdot ((1 - (-B + D) / C)^{n+1} - (1 - D / C)^{n+1}) \cdot 1 / 2 \cdot (X_{n2}(i+1)^2 - X_{n2}(i)^2) + (-f_{cd} + d\text{Sigma}) \cdot B / 2 \cdot (X_{n2}(i+1)^2 - X_{n2}(i)^2) \text{ if } A = 0 \text{ or}$$

$$MS_{yn2,zn2_i} = f_{cd} \cdot C^2 / ((n+1) \cdot (n+2) \cdot A) \cdot (X_{n2}(i+1) \cdot (1 - (-Y_{n2}(i+1) + D) / C)^{n+2} - X_{n2}(i) \cdot (1 - (-Y_{n2}(i) + D) / C)^{n+2} - C / (n+3) / A \cdot ((1 - (-Y_{n2}(i+1) + D) / C)^{n+3} - (1 - (-Y_{n2}(i) + D) / C)^{n+3})) - 0.5 \cdot (X_{n2}(i+1)^2 - X_{n2}(i)^2) \cdot f_{cd} \cdot C / (n+1) \cdot (1 - D / C)^{n+1} + (-f_{cd} + d\text{Sigma}) \cdot (A / 3 \cdot (X_{n2}(i+1)^3 - X_{n2}(i)^3) + B / 2 \cdot (X_{n2}(i+1)^2 - X_{n2}(i)^2)) \text{ if } A \neq 0$$

$$MS_{yn2,zn2} = \text{sum}(MS_{yn2,zn2_i})$$

$$MS_{xn2,zn2_i} = (f_{cd} \cdot C / (n+1) \cdot (B \cdot (1 - (-B + D) / C)^{n+1} - C / (n+2) \cdot ((1 - (-B + D) / C)^{n+2} - (1 - D / C)^{n+2})) + (-f_{cd} + d\text{Sigma}) / 2 \cdot B^2) \cdot (X_{n2}(i+1) - X_{n2}(i)) \text{ if } A = 0 \text{ or}$$

$$MS_{xn2,zn2_i} = f_{cd} \cdot C^2 / ((n+1) \cdot (n+2) \cdot A) \cdot (Y_{n2}(i+1) \cdot (1 - (-Y_{n2}(i+1) + D) / C)^{n+2} - Y_{n2}(i) \cdot (1 - (-Y_{n2}(i) + D) / C)^{n+2} - C / (n+3) \cdot (2 \cdot (1 - (-Y_{n2}(i+1) + D) / C)^{n+3} - 2 \cdot (1 - (-Y_{n2}(i) + D) / C)^{n+3}) + A \cdot (1 - D / C)^{n+2} \cdot dx) + (d\text{Sigma} - f_{cd}) \cdot (A^2 / 6 \cdot (X_{n2}(i+1)^3 - X_{n2}(i)^3) + A \cdot B / 2 \cdot (X_{n2}(i+1)^2 - X_{n2}(i)^2) + B^2 / 2 \cdot dx) \text{ if } A \neq 0$$

$$MS_{xn2,zn2} = \text{sum}(MS_{xn2,zn2_i})$$

$$X_{c2}' = MS_{yn2,zn2} / N_{c2}$$

$$Y_{c2}' = MS_{xn2,zn2} / N_{c2}$$

$$X_{c2} = X_{n2c} + \cos(\text{Angle} - \pi / 2) \cdot X_{c2}' - \sin(\text{Angle} - \pi / 2) \cdot Y_{c2}'$$

$$Y_{c2} = Y_{n2c} + \sin(\text{Angle} - \pi / 2) \cdot X_{c2}' + \cos(\text{Angle} - \pi / 2) \cdot Y_{c2}'$$

$$N_{c2} = 0.867857e6$$

$$N_{c'} = N_{c1} - N_{c2} = 1.301786e6$$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$$X_{c1} = 0$$

$$Y_{c1} = \text{delta} + y_b / 2 = 0.171428571$$

$$X_{c2} = 0$$

$$Y_{c2} = 0.117857143$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$X_{cg} = (X_{c1} \cdot N_{c1} - X_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2}) = 0.000000$$

$$Y_{cg} = (Y_{c1} \cdot N_{c1} - Y_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2}) = 0.207143$$

Internal forces in concrete:

$$N_c = N_{c'} = 1301.79e3$$

$$M_{xc} = -N_c \cdot Y_{cg} = -269.66e3$$

$$M_{yc} = N_{c'} \cdot X_{cg} = 0.00e3$$

- Results for reduced section:

$$N = N_c + N_s = \underline{752.22e3}$$

$$M_x = M_{xc} + M_{xs} = \underline{-407.05e3}$$

$$M_y = M_{yc} + M_{ys} = \underline{0.00e3}$$

7 Case 6: Calculation of internal forces for given state of strain in the section and power-rectangular model of concrete

7.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, extreme strains in the section and give internal forces.

7.2 Summary:

This sample gives as results the internal forces for:

- A rectangular concrete shape section with symmetric reinforcement (Case 6a and Case 6b)
- A rectangular concrete shape section with asymmetric reinforcement (Case 6c)
- A power-rectangular model of concrete and horizontal branch model of steel
- A section under unidirectional bending with entire section under compression (Case 6a)
- A section under unidirectional bending with a part of section under compression (Case 6b and Case 6c)
- A state of strain with ultimate compressive strain in concrete (Case 6a and Case 6b)
- A strain below the ultimate compressive strain in concrete (Case 6c).

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel.

Based on input data, Component gives as output results:

- Results for reinforcement bars:
 - Axial force (N_s)
 - Moment with respect to x_{cc} axis (M_{xs})
 - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
 - Area of contour under compressive stress (A_c)
 - Center of gravity of stresses in the concrete (x_{cg} , y_{cg})
 - Axial force (N_c)
 - Moment with respect to x_{cc} axis (M_{xc})
 - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
 - Axial force (N)
 - Moment with respect to x_{cc} axis (M_x)
 - Moment with respect to y_{cc} axis (M_y)

For these results, x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

7.3 Case 6a

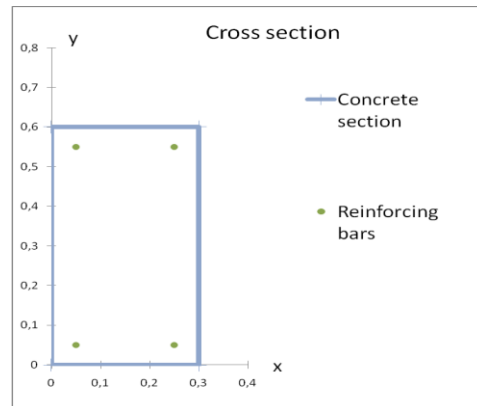
7.3.1 Data:

- Geometry:
 - Concrete section:

$x = 0.0$ $y = 0.0$
 $x = 0.0$ $y = 0.6$
 $x = 0.3$ $y = 0.6$
 $x = 0.3$ $y = 0.0$

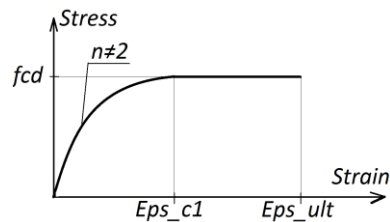
○ Reinforcing bars:

$x = 0.05$ $y = 0.05$ $\phi = 0.012$
 $x = 0.25$ $y = 0.05$ $\phi = 0.012$
 $x = 0.25$ $y = 0.55$ $\phi = 0.012$
 $x = 0.05$ $y = 0.55$ $\phi = 0.012$



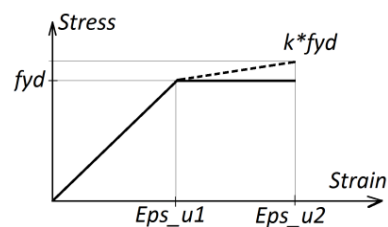
• Concrete parameters:

- Design strength $f_{cd} = 30e6$
- Modulus of elasticity $E_c = 32e9$
- Strain-stress model: power-rectangular
- Power: $n = 1.4$
- Strain ultimate limit $Eps_ult = 0.0035$
- Strain relation change over $Eps_c1 = 0.0020$

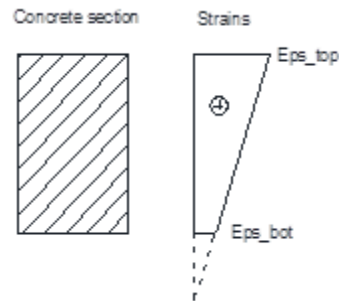


• Steel parameters:

- Design strength $f_{yd} = 400e6$
- Hardening factor $k = 1.0$
- Modulus of elasticity $E_s = 200e9$
- Strain ultimate limit $Eps_u2 = 0.1$



- Strains:
 - Top strain $Eps_top = 0.0035$
 - Bottom strain $Eps_bot = 0.0005$
 - Neutral axis angle $Angle = \pi \cdot 3 / 2$



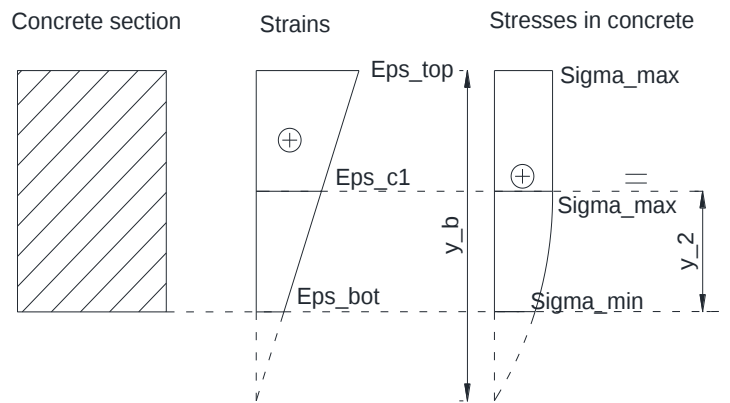
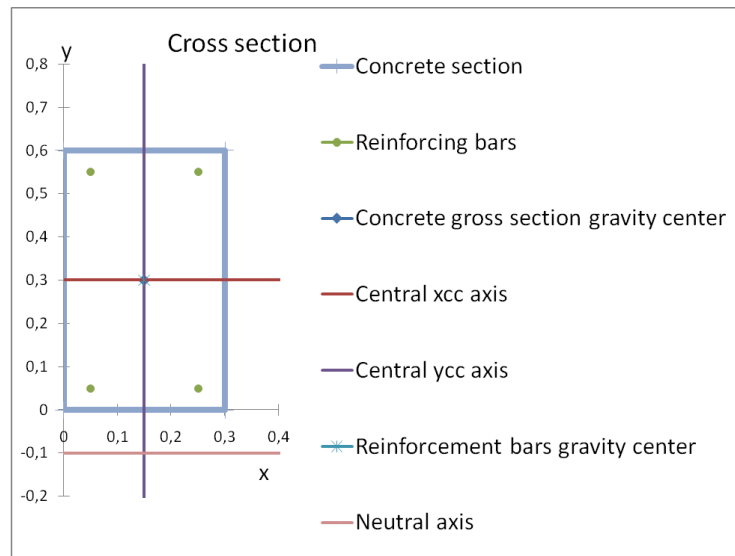
7.3.2 Search for:

- Results for reinforcement bars: N_s , M_{xs} , M_{ys}
- Results for concrete section: A_c , x_{cg} , y_{cg} , N_c , M_{xc} , M_{yc}
- Results for reduced section: N , M_x , M_y

7.3.3 Results from Component

- Results for reinforcement bars:
 - $N_s = 221.17e3$
 - $M_{xs} = -25.13e3$
 - $M_{ys} = 0.00e3$
- Results for concrete section:
 - $A_c = 0.1800$
 - $x_{cg} = 0.00000000$
 - $y_{cg} = 0.03405827$
 - $N_c = 4630.08e3$
 - $M_{xc} = -157.69e3$
 - $M_{yc} = 0.00e3$
- Results for reduced section:
 - $N = 4851.25e3$
 - $M_x = -182.83e3$
 - $M_y = 0.00e3$

7.3.4 Results (manual calculations):



- Results for reinforcement bars:
Distance between maximum compression corner of the section and neutral axis:
height of a section: $h = 0.6$
 $y_b = -Eps_top \cdot h / (Eps_bot - Eps_top) = 0.7$
 $Eps_u1 = f_{yd} / E_s$

Concrete gross section gravity center:

$$x_{Cc} = 0.15$$

$$y_{Cc} = 0.30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar i:

$$\text{If } |eps_i| \leq f_{yd} / E_s \text{ then} \\ \sigma_i = eps_i / Eps_u1 \cdot f_{yd}$$

else

$$\sigma_i = f_{yd} + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) \cdot (k - 1) \cdot f_{yd}$$

i	x	y	ϕ	Distance d from neutral axis	strain eps = d / y_b • Eps_top	stresses sigma / 10e6	force in bar N = sigma • π • $\phi^2 / 4$	Positions in central axes coordinate system x _{cc} y _{cc}		M _{xs} = - N • y _{cc}	M _{ys} = N • x _{cc}
1	0,05	0,05	0,016	0,150	0,00075	150	30159	-0,10	-0,25	7540	-3016
2	0,25	0,05	0,016	0,150	0,00075	150	30159	0,10	-0,25	7540	3016
3	0,25	0,55	0,016	0,650	0,00325	400	80425	0,10	0,25	-20106	8042
4	0,05	0,55	0,016	0,650	0,00325	400	80425	-0,10	0,25	-20106	-8042
						Sum of forces in bars:	221168			-25133	0

$$N_s = 221.17e3$$

$$M_{xs} = -25.13e3$$

$$M_{ys} = 0.00e3$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

$$\text{if } \text{eps} \leq \text{Eps_c1} \text{ then } N_{sc} = f_{cd} \cdot (1 - (1 - \text{eps} / \text{Eps_c1})^n) \cdot \text{Area}$$

$$\text{if } \text{eps} > \text{Eps_c1} \text{ then } N_{sc} = f_{cd} \cdot \text{Area}$$

i	x	y	ϕ	strain eps	Positions in central axes coordinate system x _{cc} y _{cc}		Area of concrete with compression stress cut from the section in place of bars	Force in concrete in place of bars N _{sc}	M _{xsc} = - N _{sc} • y _{cc}	M _{ysc} = N _{sc} • x _{cc}
1	0,05	0,05	0,016	0,00075	-0,1	-0,25	0,000201062	2908	727	-291
2	0,25	0,05	0,016	0,00075	0,1	-0,25	0,000201062	2908	727	291
3	0,25	0,55	0,016	0,00325	0,1	0,25	0,000201062	6032	-1508	603
4	0,05	0,55	0,016	0,00325	-0,1	0,25	0,000201062	6032	-1508	-603
							Sum of forces:	17880	-1562	0

Reduction forces in concrete:

$$N_{sc} = 17.88e3$$

$$M_{xsc} = -1.56e3$$

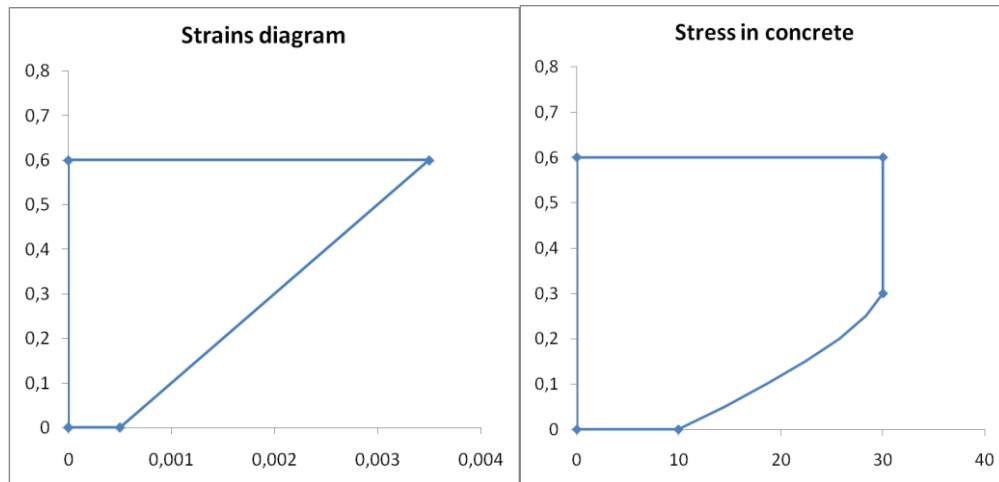
$$M_{ysc} = 0.00e3$$

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

$$x_{scg} = \text{sum}(x_{cc_i} \cdot \text{Area_i}) / \text{sum}(\text{Area_i}) = 0.00000$$

$$y_{scg} = \text{sum}(y_{cc_i} \cdot \text{Area_i}) / \text{sum}(\text{Area_i}) = 0.08736$$

- Results for concrete section:



Height of compressed part of the section:

$$h = 0.6000$$

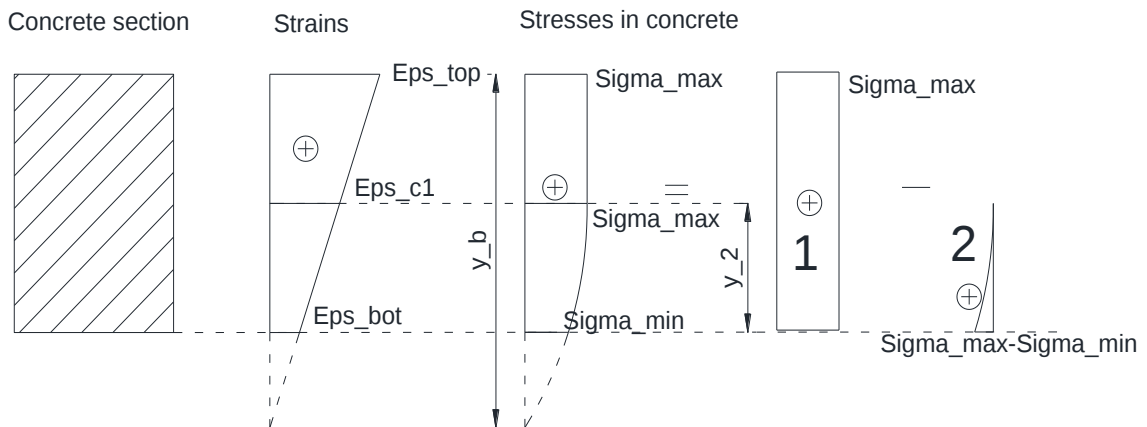
Area of contour under compressive stress:

$$A_c = 0.3 \cdot 0.6000 = 0.1800$$

Axial force N_c in compressed part of concrete section is calculated by subtraction of volume of a power ($n = 1.4$) solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of N_c in Introduction).

$$N_{c'} = N_{c1} - N_{c2}$$

$$N_c = N_{c'} - N_{sc}$$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

Moments M_{xc} and M_{yc} caused by stresses acting on entire compressed gross area are calculated following below equations:

$$M_{xc} = - N_c \cdot y_{cg}$$

$$M_{yc} = N_c \cdot x_{cg}$$

$$\text{Sigma_max} = f_{cd} = 30.000\text{e}6$$

$$\text{Sigma_min} = f_{cd} \cdot (1 - (1 - \text{Eps_bot} / \text{Eps_c1})^n) = 9.946\text{e}6$$

Distance between gravity center of contour and neutral axis:

$$\text{delta} = -(0.6 / 2 - y_b) = 0.4$$

$$\text{Angle} = 3 / 2 \cdot \pi = 4.712$$

Distance between the gravity center of contour of the section and:

- the axis with strains equal Eps_c1 if $\text{Eps_top} \geq \text{Eps_c1}$ or
- the axis parallel to neutral axis intersecting the most compressed corner of the section if

$\text{Eps_top} < \text{Eps_c1}$,:

$$\text{delta2} = -\text{delta} + \text{MIN}(y_b; \text{Eps_c1} / \text{Eps_top} \cdot y_b) = 0$$

$$\text{Angle} = 3 / 2 \cdot \pi = 4.712$$

Height of the part of the section where concrete stress is not constant:

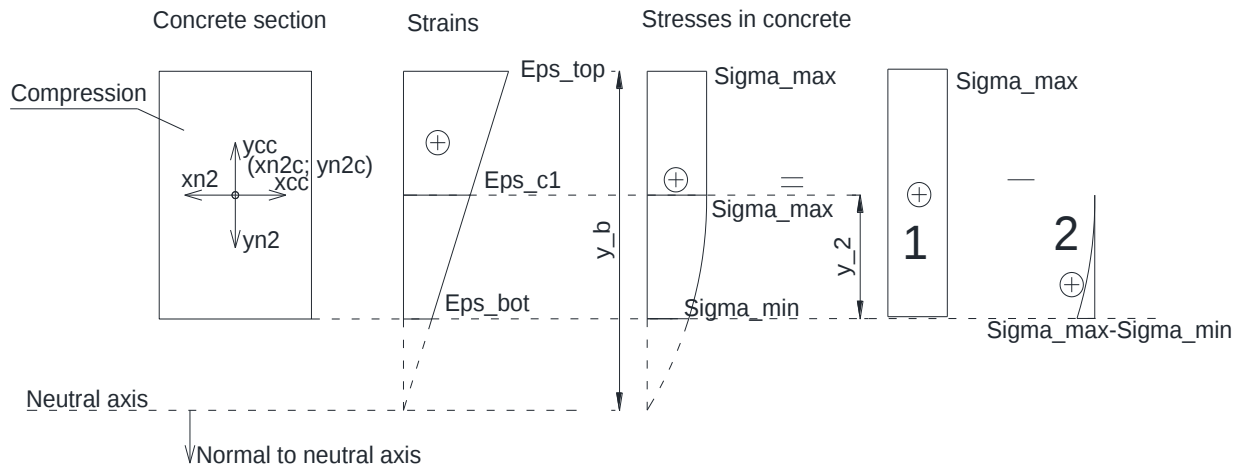
$$y_2 = 0.3 - \text{delta2} = 0.3$$

Axial force in compressed part of concrete gross section

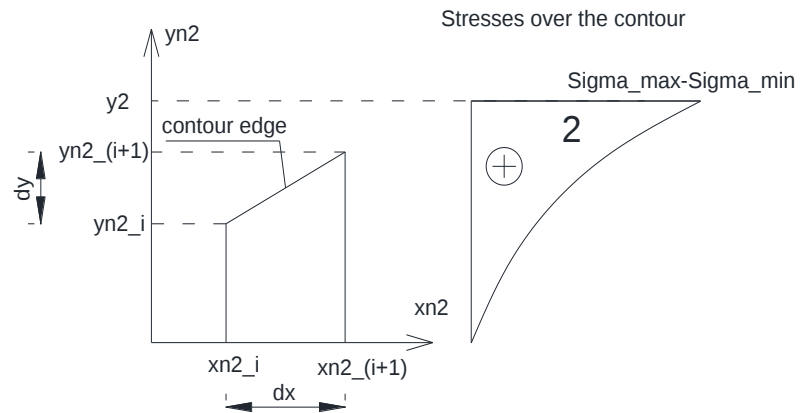
$$N_{c1} = A_c \cdot \text{Sigma_max} = 5.400\text{e}6$$

To calculate N_{c2} we need to calculate an integral of stresses over contour for which stresses are changing according to the shape 2 on picture below. To do this we define a new coordinate system x_{n2} , y_{n2} (see picture below), where x_{n2} is an axis parallel to neutral axis, in distance delta2 from the gravity center of contour of the section, and it crosses axis y_{n2} in point $(x_{n2c}; y_{n2c})$ given in central axis coordinate system.

$$(x_{n2c}; y_{n2c}) = (0; \text{delta2})$$



Now we find coordinates of contour under stress solid 2 in (x_{n2}, y_{n2}) coordinates system and calculate an integral of stresses over this contour.



i	Coordinates in (x,y) system		Coordinates in central axes system		Coordinates in (x _{n2} , y _{n2}) system		dx	dy	A	B	N _{c2}	Static moment of stress solid	
	x	y	x _{cc}	y _{cc}	x _{n2}	y _{n2}						MS _{yn2,zn2}	MS _{xn2,zn2}
1	0,00	0,00	-0,15	-0,30	0,15	0,30	0,00	-0,30	0,00	0,00	0,00	0,00	0,00
2	0,00	0,30	-0,15	0,00	0,15	0,00	-0,30	0,00	0,00	0,00	0,00	0,00	0,00
3	0,30	0,30	0,15	0,00	-0,15	0,00	0,00	0,30	0,00	0,00	0,00	0,00	0,00
4	0,30	0,00	0,15	-0,30	-0,15	0,30	0,30	0,00	0,00	0,30	0,75	0,00	0,16
5	0,00	0,00	-0,15	-0,30	0,15	0,30	0,00	0,00	0,00	0,00	0,00	0,00	0,00
	N _{c2} = sum(N _{c2 i}) =											0,75	
	C=									0,40			
	D=									0,40			
gravity center of stress solid 2 in (x _{n2} ; y _{n2}) coordinates system												x _{c2} '	0,0000
												y _{c2} '	0,2118
gravity center of stress solid 2 in central axes (x _{cc} ; y _{cc}) coordinates system												x _{c2}	0,0000
												y _{c2}	-0,2118

Where:

$$x_{cc} = x - x_{Cc}$$

$$y_{cc} = y - y_{Cc}$$

$$x_{n2} = \cos(\text{Angle} - \pi / 2) \cdot (x_{cc} - x_{n2c}) + \sin(\text{Angle} - \pi / 2) \cdot (y_{cc} - y_{n2c})$$

$$y_{n2} = -\sin(\text{Angle} - \pi / 2) \cdot (x_{cc} - x_{n2c}) + \cos(\text{Angle} - \pi / 2) \cdot (y_{cc} - y_{n2c})$$

$$dx = x_{i+1} - x_i$$

$$dy = y_{i+1} - y_i$$

$$A = dy/dx \text{ if } dx \neq 0 \text{ or } A=0 \text{ if } dx=0$$

$$B = -dy/dx \cdot x_{n2_{i+1}} + y_{n2_{i+1}} \text{ if } dx \neq 0 \text{ or } B = 0 \text{ if } dx = 0$$

$$C = y_b \cdot \text{Eps}_{c1} / \text{Eps}_{top}$$

$$D = \text{MIN}(\text{Eps}_{c1}; \text{Eps}_{top}) \cdot y_b / \text{Eps}_{top}$$

$$n = 1.4 \text{ (power of stress-strain relation curve)}$$

$$d\text{Sigma} = \text{Sigma}_{max} - \text{Sigma}_{min}$$

$$N_{c2_i} = 0 \text{ if } A = 0 \text{ and } B = 0 \text{ or}$$

$$N_{c2_i} = f_{cd} \cdot C / (n+1) \cdot (C / (n+2) / A \cdot ((1 - (-y_{n2_{i+1}} + D) / C)^{n+2} - (1 - (-y_{n2_i} + D) / C)^{n+2}) - (x_{n2_{i+1}} - x_{n2_i}) \cdot (1 - D / C)^{n+1}) + (d\text{Sigma} - f_{cd}) \cdot (A / 2 \cdot (x_{n2_{i+1}}^2 - x_{n2_i}^2) + B \cdot (x_{n2_{i+1}} - x_{n2_i})) \text{ if } A \neq 0 \text{ or}$$

$$N_{c2_i} = (f_{cd} \cdot C / (n+1) \cdot ((1 - (-B + D) / C)^{n+1} - (1 - D / C)^{n+1})) \cdot dx + (-f_{cd} + d\text{Sigma}) \cdot B \cdot dx \text{ if } A = 0 \text{ and } B \neq 0$$

$$N_{c2} = \text{sum}(N_{c2_i})$$

$$MS_{yn2,zn2_i} = f_{cd} \cdot C / (n+1) \cdot ((1 - (-B + D) / C)^{n+1} - (1 - D / C)^{n+1}) \cdot 1 / 2 \cdot (x_{n2_{i+1}}^2 - x_{n2_i}^2) + (-f_{cd} + d\text{Sigma}) \cdot B / 2 \cdot (x_{n2_{i+1}}^2 - x_{n2_i}^2) \text{ if } A = 0 \text{ or}$$

$$MS_{yn2,zn2_i} = f_{cd} \cdot C^2 / ((n+1) \cdot (n+2) \cdot A) \cdot (x_{n2_{i+1}} \cdot (1 - (-y_{n2_{i+1}} + D) / C)^{n+2} - x_{n2_i} \cdot (1 - (-y_{n2_i} + D) / C)^{n+2} - C / (n+3) / A \cdot ((1 - (-y_{n2_{i+1}} + D) / C)^{n+3} - (1 - (-y_{n2_i} + D) / C)^{n+3})) - 0.5 \cdot (x_{n2_{i+1}}^2 - x_{n2_i}^2) \cdot f_{cd} \cdot C / (n+1) \cdot (1 - D / C)^{n+1} + (-f_{cd} + d\text{Sigma}) \cdot (A / 3 \cdot (x_{n2_{i+1}}^3 - x_{n2_i}^3) + B / 2 \cdot (x_{n2_{i+1}}^2 - x_{n2_i}^2)) \text{ if } A \neq 0$$

$$MS_{yn2,zn2} = \text{sum}(MS_{yn2,zn2_i})$$

$$MS_{xn2,zn2_i} = (f_{cd} \cdot C / (n+1) \cdot (B \cdot (1 - (-B + D) / C)^{n+1} - C / (n+2) \cdot ((1 - (-B + D) / C)^{n+2} - (1 - D / C)^{n+2})) + (-f_{cd} + d\text{Sigma}) / 2 \cdot B^2) \cdot (x_{n2_{i+1}} - x_{n2_i}) \text{ if } A = 0 \text{ or}$$

$$MS_{xn2,zn2_i} = f_{cd} \cdot C^2 / ((n+1) \cdot (n+2) \cdot A) \cdot (y_{n2_{i+1}} \cdot (1 - (-y_{n2_{i+1}} + D) / C)^{n+2} - y_{n2_i} \cdot (1 - (-y_{n2_i} + D) / C)^{n+2} - C / (n+3) \cdot (2 \cdot (1 - (-y_{n2_{i+1}} + D) / C)^{n+3} - 2 \cdot (1 - (-y_{n2_i} + D) / C)^{n+3}) + A \cdot (1 - D / C)^{n+2} \cdot dx) + (d\text{Sigma} - f_{cd}) \cdot (A^2 / 6 \cdot (x_{n2_{i+1}}^3 - x_{n2_i}^3) + A \cdot B / 2 \cdot (x_{n2_{i+1}}^2 - x_{n2_i}^2) + B^2 / 2 \cdot dx) \text{ if } A \neq 0$$

$$MS_{xn2,zn2} = \text{sum}(MS_{xn2,zn2_i})$$

$$x_{c2}' = MS_{yn2,zn2} / N_{c2}$$

$$y_{c2}' = MS_{xn2,zn2} / N_{c2}$$

$$x_{c2} = x_{n2c} + \cos(\text{Angle} - \pi / 2) \cdot x_{c2}' - \sin(\text{Angle} - \pi / 2) \cdot y_{c2}'$$

$$y_{c2} = y_{n2c} + \sin(\text{Angle} - \pi / 2) \cdot x_{c2}' + \cos(\text{Angle} - \pi / 2) \cdot y_{c2}'$$

$$N_{c2} = 0.752035e6$$

$$N_c' = N_{c1} - N_{c2} = 4.647965e6$$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$$x_{c1} = 0$$

$$y_{c1} = 0$$

$$x_{c2} = 0$$

$$y_{c2} = -0.211764706$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.000000$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.034058$$

Internal forces in concrete:

$$\begin{aligned}
 N_c &= N_{c'} - N_{sc} = \underline{4630.08e3} \\
 M_{xc} &= - N_c \cdot y_{cg} = \underline{-157.69e3} \\
 M_{yc} &= N_{c'} \cdot x_{cg} = \underline{0.00e3}
 \end{aligned}$$

Results for reduced section:

$$\begin{aligned}
 N &= N_c + N_s = \underline{4851.25e3} \\
 M_x &= M_{xc} + M_{xs} = \underline{-182.83e3} \\
 M_y &= M_{yc} + M_{ys} = \underline{0.00e3}
 \end{aligned}$$

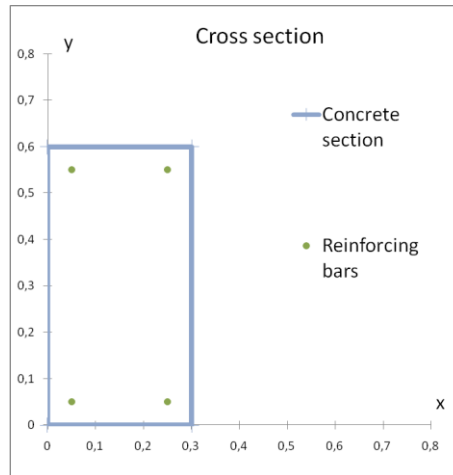
7.4 Case 6b

7.4.1 Data:

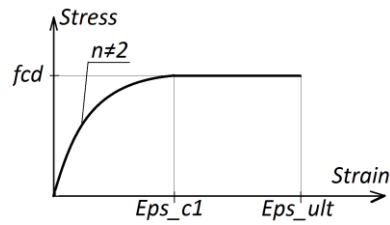
- Geometry:
 - Concrete section:

$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.6$
$x = 0.3$	$y = 0.6$
$x = 0.3$	$y = 0.0$
 - Reinforcing bars:

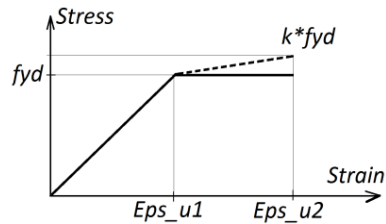
$x = 0.05$	$y = 0.05$	$\phi = 0.032$
$x = 0.25$	$y = 0.05$	$\phi = 0.032$
$x = 0.25$	$y = 0.55$	$\phi = 0.032$
$x = 0.05$	$y = 0.55$	$\phi = 0.032$



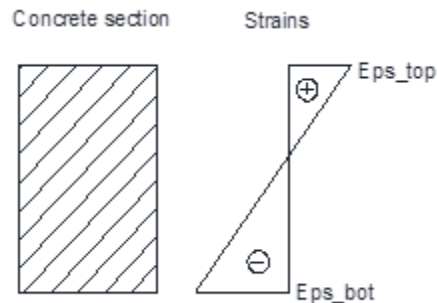
- Concrete parameters:
 - Design strength $f_{cd} = 30e6$
 - Modulus of elasticity $E_c = 32e9$
 - Strain-stress model: power-rectangular
 - Power: $n = 1.8$
 - Strain ultimate limit $Eps_ult = 0.0035$
 - Strain relation change over $Eps_c1 = 0.0020$



- Steel parameters:
 - Design strength $f_{yd} = 400e6$
 - Hardening factor $k = 1.0$
 - Modulus of elasticity $E_s = 200e9$
 - Strain ultimate limit $Eps_u2 = 0.1$



- Strains:
 - Top strain $Eps_top = 0.0035$
 - Bottom strain $Eps_bot = -0.03034666$
 - Neutral axis angle $Angle = \pi \cdot 3 / 2$



7.4.2 Search for:

- Results for reinforcement bars: N_s , M_{xs} , M_{ys}
- Results for concrete section: A_c , x_{cg} , y_{cg} , N_c , M_{xc} , M_{yc}
- Results for reduced section: N , M_x , M_y

7.4.3 Results from Component

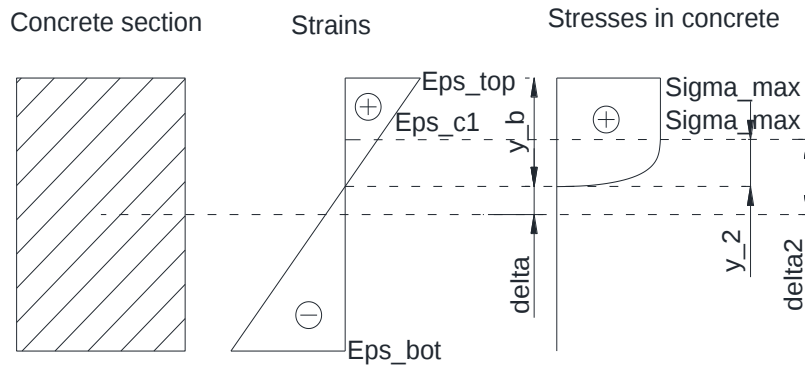
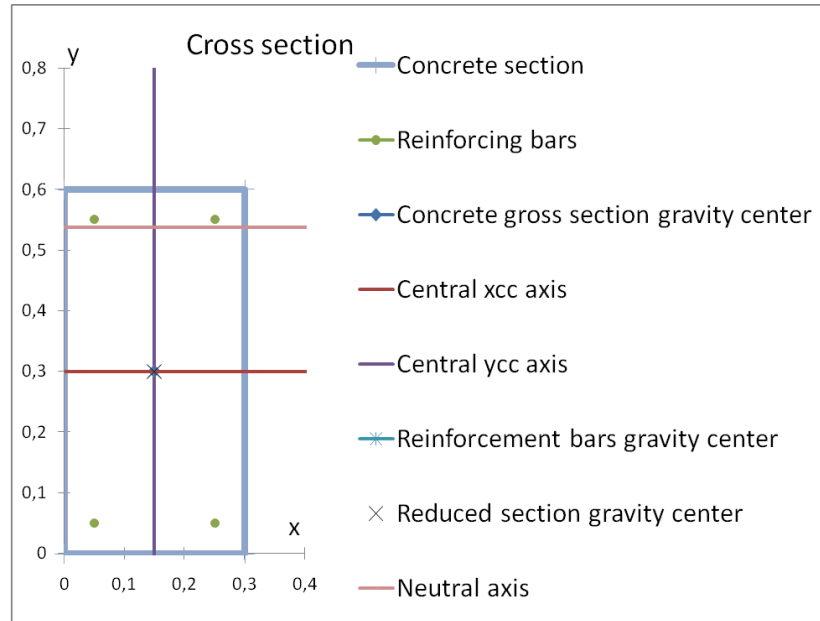
- Results for reinforcement bars:
 - $N_s = -420.68e3$
 - $M_{xs} = -216.53e3$
 - $M_{ys} = 0.00e3$
- Results for concrete section:
 - $A_c = 0.0187$
 - $x_{cg} = 0.00000000$
 - $y_{cg} = 0.27592078$

$N_c = 420.68e3$
 $M_{xc} = -116.08e3$
 $M_{yc} = 0.00e3$

- Results for reduced section:

$N = 0.00e3$
 $M_x = -332.60e3$
 $M_y = 0.00e3$

7.4.4 Results (manual calculations):



- Results for reinforcement bars:
 Distance between maximum compression corner of the section and neutral axis:
 height of a section: $h = 0.6$
 $y_b = -Eps_{top} \cdot h / (Eps_{bot} - Eps_{top}) = 0.0623$
 $Eps_{u1} = f_{yd} / E_s$

Concrete gross section gravity center:

$$x_{Cc} = 0.15$$

$$y_{Cc} = 0.30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar i:

If $|\text{eps}_i| \leq f_{yd} / E_s$ then

$$\sigma_i = \text{eps}_i / E_{s_u1} \cdot f_{yd}$$

else

$$\sigma_i = f_{yd} + (|\text{eps}_i| - E_{s_u1}) / (E_{s_u2} - E_{s_u1}) \cdot (k - 1) \cdot f_{yd}$$

i	x	y	ϕ	Distance d from neutral axis	Strain $\text{eps} = d /$ $y_b \cdot$ $E_{s_{top}}$	Stresses $\sigma_i /$ 10e6	Force in bar $N =$ $\sigma_i \cdot$ $\pi \cdot \phi^2 / 4$	Positions in central axes coordinate system x_{Cc} y_{Cc}		$M_{xs} = -N$ $\cdot y_{Cc}$	$M_{ys} = N$ $\cdot x_{Cc}$
1	0,05	0,05	0,032	-0,488	-0,02738	-400	-321699	-0,10	-0,25	-80425	32170
2	0,25	0,05	0,032	-0,488	-0,02738	-400	-321699	0,10	-0,25	-80425	-32170
3	0,25	0,55	0,032	0,012	0,00069	138	111358	0,10	0,25	-27839	11136
4	0,05	0,55	0,032	0,012	0,00069	138	111358	-0,10	0,25	-27839	-11136
						Sum of forces in bars:	-420682			-	0

$$N_s = -420.68e3$$

$$M_{xs} = -216.53e3$$

$$M_{ys} = 0.00e3$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

if $\text{eps} \leq E_{s_{c1}}$ then $N_{sc} = f_{cd} \cdot (1 - (1 - \text{eps}/E_{s_{c1}})^n) \cdot \text{Area}$

if $\text{eps} > E_{s_{c1}}$ then $N_{sc} = f_{cd} \cdot \text{Area}$

i	x	y	ϕ	strain eps	Positions in central axes coordinate system x_{Cc} y_{Cc}		Area of concrete with compression stress cut from the section in place of bars	Force in concrete in place of bars N_{sc}	$M_{xsc} = -$ $N_{sc} \cdot y_{Cc}$	$M_{ysc} =$ $N_{sc} \cdot x_{Cc}$
1	0,05	0,05	0,032	-0,02738	-0,10	-0,25				
2	0,25	0,05	0,032	-0,02738	0,10	-0,25				
3	0,25	0,55	0,032	0,00069	0,10	0,25	0,000804248	12898	-3224	1290
4	0,05	0,55	0,032	0,00069	-0,10	0,25	0,000804248	12898	-3224	-1290
							Sum of forces:	25796	-6449	0

Reduction forces in concrete:

$$N_{sc} = 25.79e3$$

$$M_{xsc} = -6.45e3$$

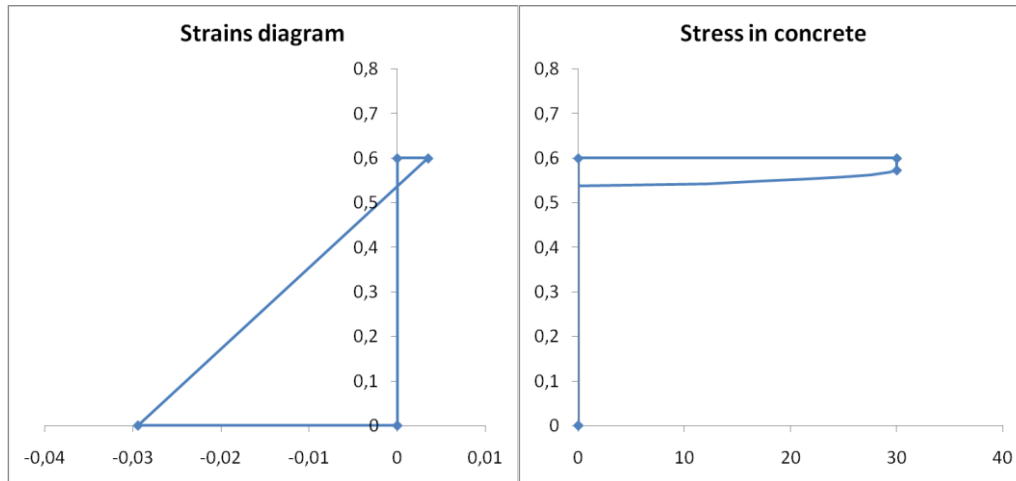
$$M_{y_{sc}} = 0.00e3$$

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

$$x_{scg} = \sum(x_{cc_i} \cdot Area_i) / \sum(Area_i) = 0.00$$

$$y_{scg} = \sum(y_{cc_i} \cdot Area_i) / \sum(Area_i) = 0.25$$

- Results for concrete section:



Height of compressed part of the section:

$$y_b = 0.0623$$

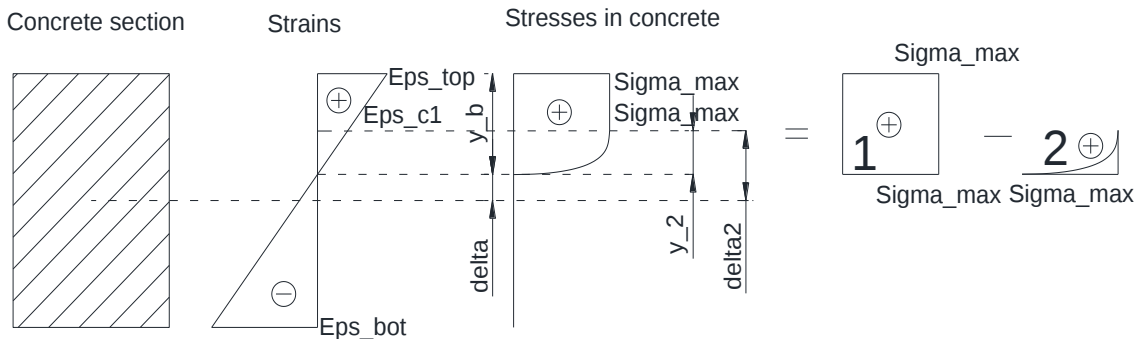
Area of contour under compressive stress:

$$A_c = 0.3 \cdot y_b = 0.0187$$

Axial force N_c in compressed part of concrete section is calculated by subtraction of volume of a power ($n = 1.8$) solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of N_c in Introduction).

$$N_{c'} = N_{c1} - N_{c2}$$

$$N_c = N_{c'} - N_{sc}$$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

Moments M_{xc} and M_{yc} caused by stresses acting on entire compressed gross area are calculated following below equations:

$$M_{xc} = - N_c \cdot y_{cg}$$

$$M_{yc} = N_c \cdot x_{cg}$$

$$\text{Sigma_max} = f_{cd} = 30.000\text{e6}$$

$$\text{Sigma_min} = 0.000\text{e6}$$

Distance between gravity center of contour and neutral axis:

$$\text{delta} = 0.6 / 2 - y_b = 0.237671$$

$$\text{Angle} = 3 / 2 \cdot \pi = 4.712$$

Distance between the gravity center of contour of the section and:

- the axis with strains equal Eps_c1 if $\text{Eps_top} \geq \text{Eps_c1}$ or

- the axis parallel to neutral axis intersecting the most compressed corner of the section if

$\text{Eps_top} < \text{Eps_c1}$,:

$$\text{delta2} = \text{delta} + \text{MIN}(y_b; \text{Eps_c1} / \text{Eps_top} \cdot y_b) = 0.273288$$

$$\text{Angle} = 3 / 2 \cdot \pi = 4.712$$

Height of the part of the section where concrete stress is not constant:

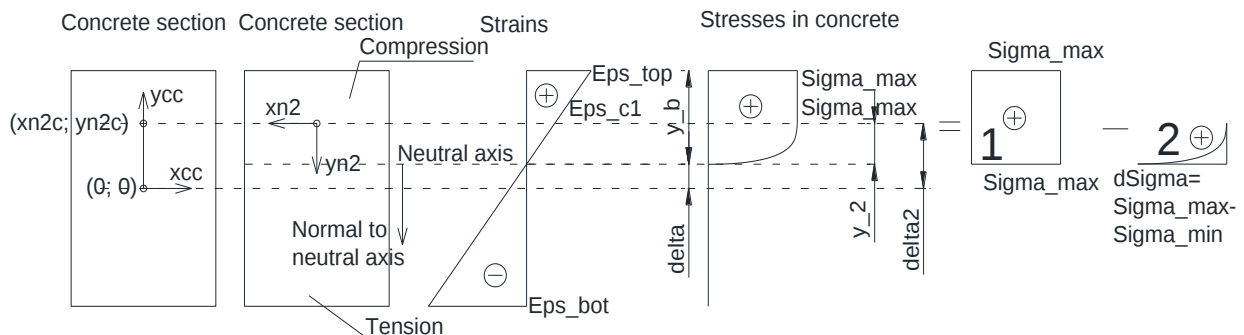
$$y_2 = \text{delta2} - \text{delta} = 0.0356165$$

Axial force in compressed part of concrete gross section

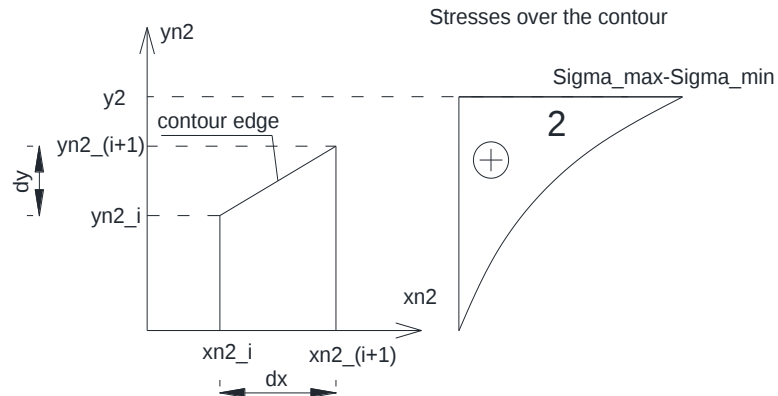
$$N_{c1} = A_c \cdot \text{Sigma_max} = 0.560960\text{e6}$$

To calculate N_{c2} we need to calculate an integral of stresses over contour for which stresses are changing according to the shape 2 on picture below. To do this we define a new coordinate system x_{n2} , y_{n2} (see picture below), where x_{n2} is an axis parallel to neutral axis, in distance delta2 from the gravity center of contour of the section, and it crosses axis y_{n2} in point $(x_{n2c}; y_{n2c})$ given in central axis coordinate system.

$$(x_{n2c}; y_{n2c}) = (0; \text{delta2})$$



Now we find coordinates of contour under stress solid 2 in (x_{n2}, y_{n2}) coordinates system and calculate an integral of stresses over this contour.



i	Coordinates in (x,y) system		Coordinates in central axes system		Coordinates in (x _{n2} , y _{n2}) system		dx	dy	A	B	N _{c2}	Static moment of stress solid		
	x	y	x _{cc}	y _{cc}	x _{n2}	y _{n2}						MS _{y_{n2},z_{n2}}	MS _{x_{n2},z_{n2}}	
1	0,30	0,57	0,15	0,27	-0,15	0,00	0,00	0,04	0,00	0,00	0,000	0,000	0,000	
2	0,30	0,54	0,15	0,24	-0,15	0,04	0,30	0,00	0,00	0,04	0,114	0,000	0,003	
3	0,00	0,54	-0,15	0,24	0,15	0,04	0,00	-0,04	0,00	0,00	0,000	0,000	0,000	
4	0,00	0,57	-0,15	0,27	0,15	0,00	-0,30	0,00	0,00	0,00	0,000	0,000	0,000	
5	0,30	0,57	0,15	0,27	-0,15	0,00	0,00	0,00	0,00	0,00	0,000	0,000	0,000	
	N _{c2} = sum(N _{c2} i) =											0,114		
	C=								0,03 6					
	D=								0,03 6					
gravity center of stress solid 2 in (x _{n2} ; y _{n2}) coordinates system												x _{c2} '	0,0000	
												y _{c2} '	0,0262	
gravity center of stress solid 2 in central axes (x _{cc} ; y _{cc}) coordinates system												x _{c2}	0,0000	
												y _{c2}	0,2470	

Where:

$$X_{cc} = X - X_{Cc}$$

$$Y_{cc} = Y - Y_{Cc}$$

$$X_{n2} = \cos(\text{Angle} - \pi / 2) \cdot (X_{cc} - X_{n2c}) + \sin(\text{Angle} - \pi / 2) \cdot (Y_{cc} - Y_{n2c})$$

$$Y_{n2} = -\sin(\text{Angle} - \pi / 2) \cdot (X_{cc} - X_{n2c}) + \cos(\text{Angle} - \pi / 2) \cdot (Y_{cc} - Y_{n2c})$$

$$dx = X_{(i+1)} - X_i$$

$$dy = Y_{(i+1)} - Y_i$$

$$A = dy/dx \text{ if } dx \neq 0 \text{ or } A=0 \text{ if } dx=0$$

$$B = -dy/dx \cdot X_{n2_{(i+1)}} + Y_{n2_{(i+1)}} \text{ if } dx \neq 0 \text{ or } B = 0 \text{ if } dx = 0$$

$$C = Y_b \cdot Eps_{c1} / Eps_{top}$$

$$D = \text{MIN}(Eps_{c1}; Eps_{top}) \cdot Y_b / Eps_{top}$$

$$n = 1.8 \text{ (power of stress-strain relation curve)}$$

$$d\text{Sigma} = \text{Sigma}_{max} - \text{Sigma}_{min}$$

$$N_{c2_i} = 0 \text{ if } A = 0 \text{ and } B = 0 \text{ or}$$

$$N_{c2_i} = f_{cd} \cdot C / (n+1) \cdot (C / (n+2) / A \cdot ((1 - (-Y_{n2_{(i+1)}} + D) / C)^{n+2} - (1 - (-Y_{n2_i} + D) / C)^{n+2}) - (X_{n2_{(i+1)}} - X_{n2_i}) \cdot (1 - D / C)^{n+1}) + (d\text{Sigma} - f_{cd}) \cdot (A / 2 \cdot (X_{n2_{(i+1)}}^2 - X_{n2_i}^2) + B \cdot (X_{n2_{(i+1)}} - X_{n2_i})) \text{ if } A \neq 0 \text{ or}$$

$$N_{c2_i} = (f_{cd} \cdot C / (n+1) \cdot ((1 - (-B + D) / C)^{n+1} - (1 - D / C)^{n+1})) \cdot dx + (-f_{cd} + d\text{Sigma}) \cdot B \cdot dx \text{ if } A = 0 \text{ and } B \neq 0$$

$$N_{c2} = \text{sum}(N_{c2_i})$$

$$MS_{Y_{n2}, Z_{n2}} = f_{cd} \cdot C / (n+1) \cdot ((1 - (-B + D) / C)^{n+1} - (1 - D / C)^{n+1}) \cdot 1 / 2 \cdot (X_{n2_{(i+1)}}^2 - X_{n2_i}^2) + (-f_{cd} + d\text{Sigma}) \cdot B / 2 \cdot (X_{n2_{(i+1)}}^2 - X_{n2_i}^2) \text{ if } A = 0 \text{ or}$$

$$MS_{Y_{n2}, Z_{n2}} = f_{cd} \cdot C^2 / ((n+1) \cdot (n+2) \cdot A) \cdot (X_{n2_{(i+1)}} \cdot (1 - (-Y_{n2_{(i+1)}} + D) / C)^{n+2} - X_{n2_i} \cdot (1 - (-Y_{n2_i} + D) / C)^{n+2} - C / (n+3) / A \cdot ((1 - (-Y_{n2_{(i+1)}} + D) / C)^{n+3} - (1 - (-Y_{n2_i} + D) / C)^{n+3})) - 0.5 \cdot (X_{n2_{(i+1)}}^2 - X_{n2_i}^2) \cdot f_{cd} \cdot C / (n+1) \cdot (1 - D / C)^{n+1} + (-f_{cd} + d\text{Sigma}) \cdot (A / 3 \cdot (X_{n2_{(i+1)}}^3 - X_{n2_i}^3) + B / 2 \cdot (X_{n2_{(i+1)}}^2 - X_{n2_i}^2)) \text{ if } A \neq 0$$

$$MS_{Y_{n2}, Z_{n2}} = \text{sum}(MS_{Y_{n2}, Z_{n2}})$$

$$MS_{X_{n2}, Z_{n2}} = (f_{cd} \cdot C / (n+1) \cdot (B \cdot (1 - (-B + D) / C)^{n+1} - C / (n+2) \cdot ((1 - (-B + D) / C)^{n+2} - (1 - D / C)^{n+2})) + (-f_{cd} + d\text{Sigma}) / 2 \cdot B^2) \cdot (X_{n2_{(i+1)}} - X_{n2_i}) \text{ if } A = 0 \text{ or}$$

$$MS_{X_{n2}, Z_{n2}} = f_{cd} \cdot C^2 / ((n+1) \cdot (n+2) \cdot A) \cdot (Y_{n2_{(i+1)}} \cdot (1 - (-Y_{n2_{(i+1)}} + D) / C)^{n+2} - Y_{n2_i} \cdot (1 - (-Y_{n2_i} + D) / C)^{n+2} - C / (n+3) \cdot (2 \cdot (1 - (-Y_{n2_{(i+1)}} + D) / C)^{n+3} - 2 \cdot (1 - (-Y_{n2_i} + D) / C)^{n+3}) + A \cdot (1 - D / C)^{n+2} \cdot dx) + (d\text{Sigma} - f_{cd}) \cdot (A^2 / 6 \cdot (X_{n2_{(i+1)}}^3 - X_{n2_i}^3) + A \cdot B / 2 \cdot (X_{n2_{(i+1)}}^2 - X_{n2_i}^2) + B^2 / 2 \cdot dx) \text{ if } A \neq 0$$

$$MS_{X_{n2}, Z_{n2}} = \text{sum}(MS_{X_{n2}, Z_{n2}})$$

$$X_{c2}' = MS_{Y_{n2}, Z_{n2}} / N_{c2}$$

$$Y_{c2}' = MS_{X_{n2}, Z_{n2}} / N_{c2}$$

$$X_{c2} = X_{n2c} + \cos(\text{Angle} - \pi / 2) \cdot X_{c2}' - \sin(\text{Angle} - \pi / 2) \cdot Y_{c2}'$$

$$Y_{c2} = Y_{n2c} + \sin(\text{Angle} - \pi / 2) \cdot X_{c2}' + \cos(\text{Angle} - \pi / 2) \cdot Y_{c2}'$$

$$N_{c2} = 0.114482e6$$

$$N_{c'} = N_{c1} - N_{c2} = 0.446478e6$$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$$X_{c1} = 0$$

$$Y_{c1} = \text{delta} + Y_b / 2 = 0.268836$$

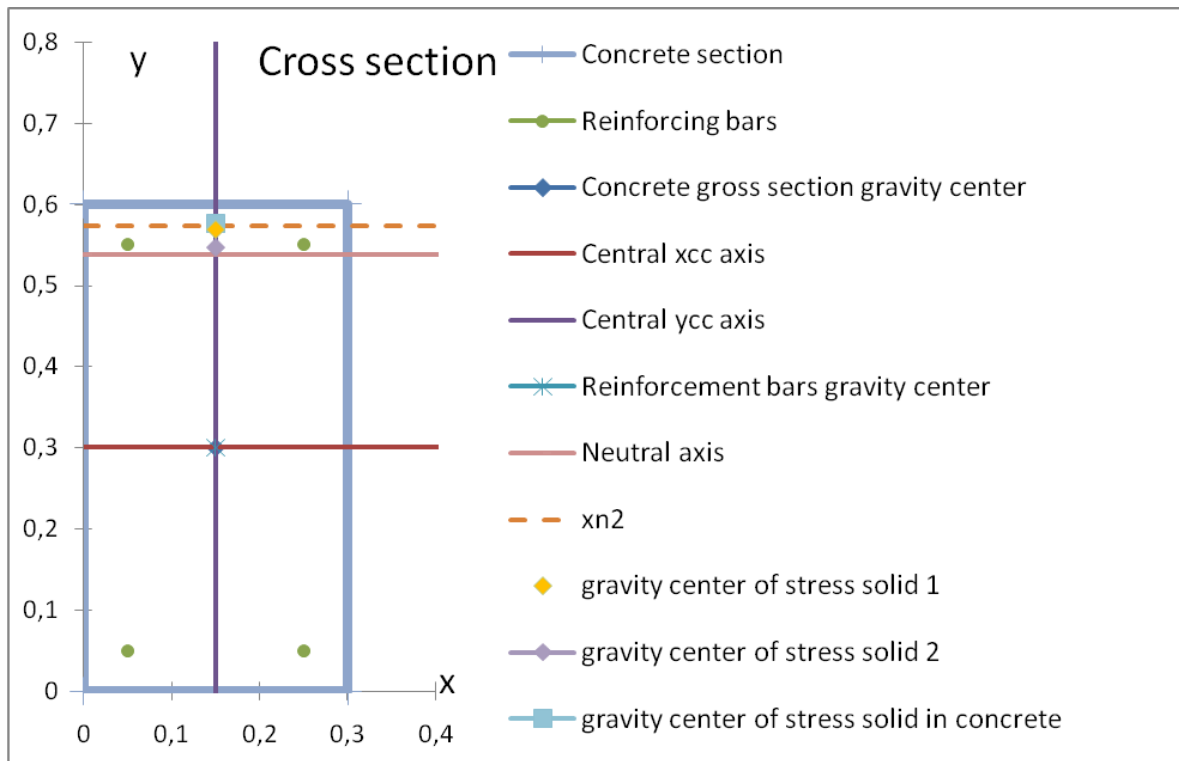
$$X_{c2} = 0$$

$$Y_{c2} = 0.247043919$$

Center of gravity of stresses in concrete in central axes coordinate system:

$$X_{cg} = (X_{c1} \cdot N_{c1} - X_{c2} \cdot N_{c2} - X_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.000000$$

$$Y_{cg} = (Y_{c1} \cdot N_{c1} - Y_{c2} \cdot N_{c2} - Y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.275921$$



Internal forces in concrete:

$$N_c = N_{c'} - N_{sc} = 420.68e3$$

$$M_{xc} = -N_c \cdot y_{cg} = -116.08e3$$

$$M_{yc} = N_{c'} \cdot x_{cg} = 0.00e3$$

- Results for reduced section:

$$N = N_c + N_s = 0.00e3$$

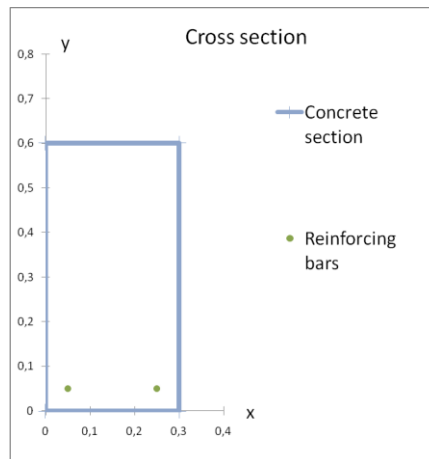
$$M_x = M_{xc} + M_{xs} = -332.60e3$$

$$M_y = M_{yc} + M_{ys} = M_{yc} + M_{ys} = 0.00e3$$

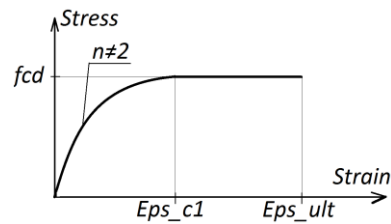
7.5 Case 6c

7.5.1 Data:

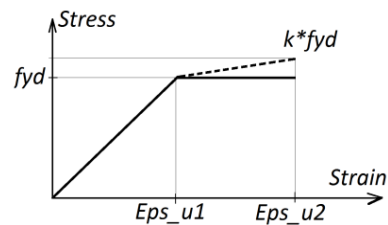
- Geometry:
 - Concrete section:
 - $x = 0.0$ $y = 0.0$
 - $x = 0.0$ $y = 0.6$
 - $x = 0.3$ $y = 0.6$
 - $x = 0.3$ $y = 0.0$
 - Reinforcing bars:
 - $x = 0.05$ $y = 0.05$ $\phi = 0.032$
 - $x = 0.25$ $y = 0.05$ $\phi = 0.032$



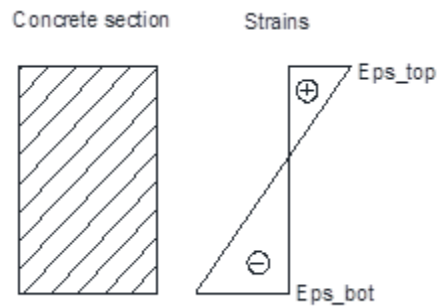
- Concrete parameters:
 - Design strength $f_{cd} = 30e6$
 - Modulus of elasticity $E_c = 32e9$
 - Strain-stress model: power-rectangular
 - Power: $n = 1.2$
 - Strain ultimate limit $Eps_ult = 0.0035$
 - Strain relation change over $Eps_c1 = 0.0020$



- Steel parameters:
 - Design strength $f_{yd} = 400e6$
 - Hardening factor $k = 1.0$
 - Modulus of elasticity $E_s = 200e9$
 - Strain ultimate limit $Eps_u2 = 0.1$



- Strains:
 - Top strain $Eps_top = 0.0015$
 - Bottom strain $Eps_bot = -0.002$
 - Neutral axis angle $Angle = \pi \cdot 3 / 2$



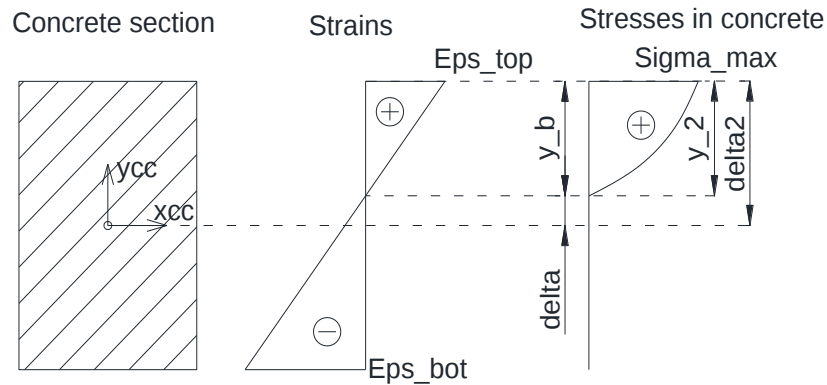
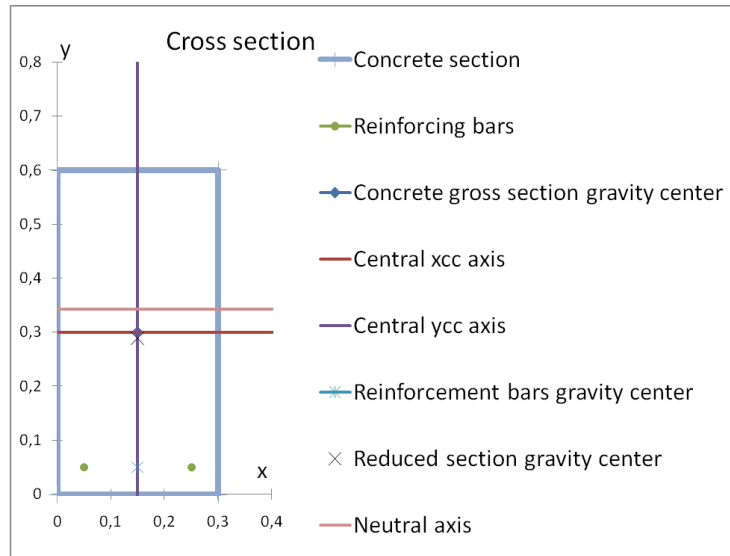
7.5.2 Search for:

- Results for reinforcement bars: N_s , M_{xs} , M_{ys}
- Results for concrete section: A_c , x_{cg} , y_{cg} , N_c , M_{xc} , M_{yc}
- Results for reduced section: N , M_x , M_y

7.5.3 Results from Component

- Results for reinforcement bars:
 $N_s = -549.57e3$
 $M_{xs} = -137.39e3$
 $M_{ys} = 0.00e3$
- Results for concrete section:
 $A_c = 0.0771$
 $x_{cg} = 0.000000$
 $y_{cg} = 0.21270852$
 $N_c = 978.12e3$
 $M_{xc} = -208.06e3$
 $M_{yc} = 0.00e3$
- Results for reduced section:
 $N = 428.55e3$
 $M_x = -345.45e3$
 $M_y = 0.00e3$

7.5.4 Results (manual calculations):



- Results for reinforcement bars:
Distance between maximum compression corner of the section and neutral axis:
height of a section: $h = 0.6$
 $y_b = -Eps_top \cdot h / (Eps_bot - Eps_top) = 0.257143$
 $Eps_u1 = f_{yd} / E_s$

Concrete gross section gravity center:

$$x_{Cc} = 0.15$$

$$y_{Cc} = 0.30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar i:

$$\text{If } |eps_i| \leq f_{yd} / E_s \text{ then}$$

$$\sigma_i = eps_i / Eps_u1 \cdot f_{yd}$$

else

$$\sigma_i = f_{yd} + (|\epsilon_i| - \epsilon_{u1}) / (\epsilon_{u2} - \epsilon_{u1}) \cdot (k - 1) \cdot f_{yd}$$

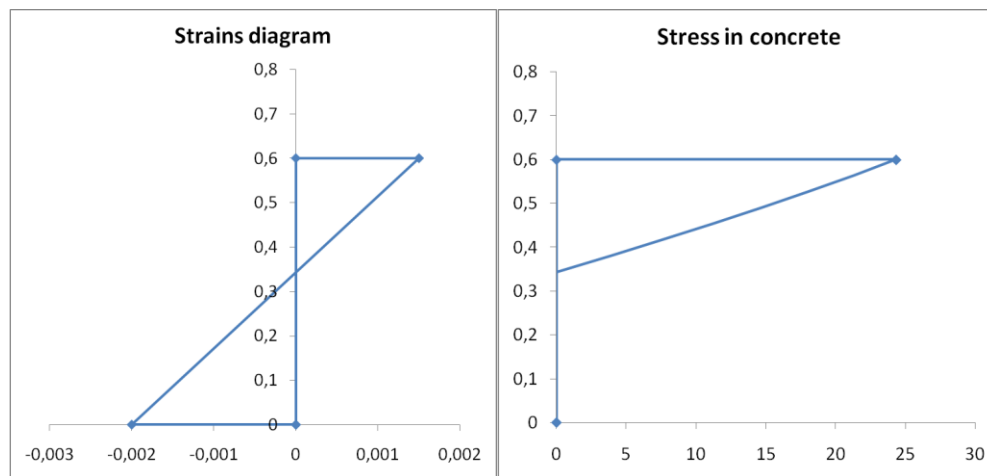
i	x	y	ϕ	Distance d from neutral axis	Strain $\epsilon = d /$ $y_b \cdot$ ϵ_{top}	Stresses $\sigma /$ $10e6$	Force in bar $N =$ $\sigma \cdot$ $\pi \cdot \phi^2 / 4$	Positions in central axes coordinate system x_{cc} y_{cc}		$M_{xs} = -N$ $\cdot y_{cc}$	$M_{ys} = N$ $\cdot x_{cc}$
1	0,05	0,05	0,032	-0,293	-0,00171	-342	-274785	-0,10	-0,25	-68696	27478
2	0,25	0,05	0,032	-0,293	-0,00171	-342	-274785	0,10	-0,25	-68696	-27478
Sum of forces in bars:							-549569			-	0

$$N_s = -549.569e3$$

$$M_{xs} = -137.392e3$$

$$M_{ys} = 0.00e3$$

- Results for concrete section:



Height of compressed part of the section:

$$y_b = 0.257143$$

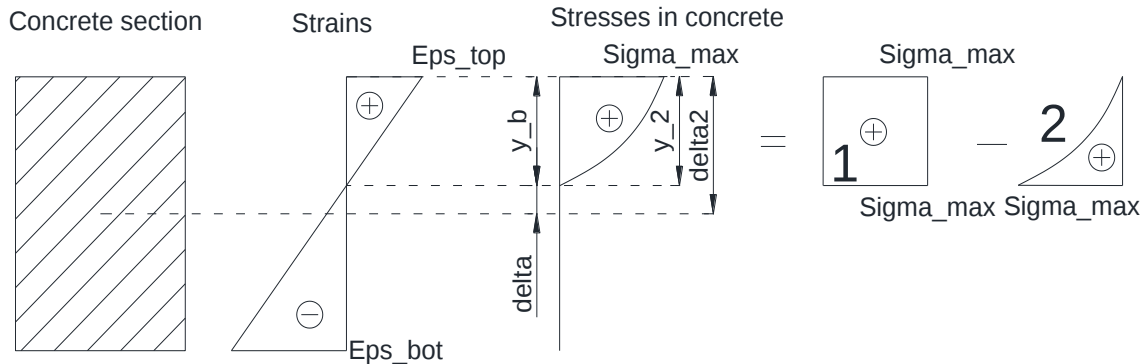
Area of contour under compressive stress:

$$A_c = 0.3 \cdot y_b = 0.0771$$

Axial force N_c in compressed part of concrete section is calculated by subtraction of volume of a power ($n = 1.2$) solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of N_c in Introduction).

$$N_c' = N_{c1} - N_{c2}$$

$$N_c = N_c'$$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as weights.

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2})$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2}) / (N_{c1} - N_{c2})$$

Moments M_{xc} and M_{yc} caused by stresses acting on entire compressed gross area are calculated following below equations:

$$M_{xc} = - N_c \cdot y_{cg}$$

$$M_{yc} = N_c \cdot x_{cg}$$

$$\text{Sigma_max} = f_{cd} \cdot (1 - (1 - \text{Eps_top} / \text{Eps_c1})^{1.2}) = 24.316\text{e6}$$

$$\text{Sigma_min} = 0.000\text{e6}$$

Distance between gravity center of contour and neutral axis:

$$\text{delta} = 0.6 / 2 - y_b = 0.042857$$

$$\text{Angle} = 3 / 2 \cdot \pi = 4.712$$

Distance between the gravity center of contour of the section and:

- the axis with strains equal Eps_c1 if $\text{Eps_top} \geq \text{Eps_c1}$ or

- the axis parallel to neutral axis intersecting the most compressed corner of the section if

$\text{Eps_top} < \text{Eps_c1}$:

$$\text{delta2} = \text{delta} + \text{MIN}(y_b; \text{Eps_c1} / \text{Eps_top} \cdot y_b) = 0.3000$$

$$\text{Angle} = 3 / 2 \cdot \pi = 4.712$$

Height of the part of the section where concrete stress is not constant:

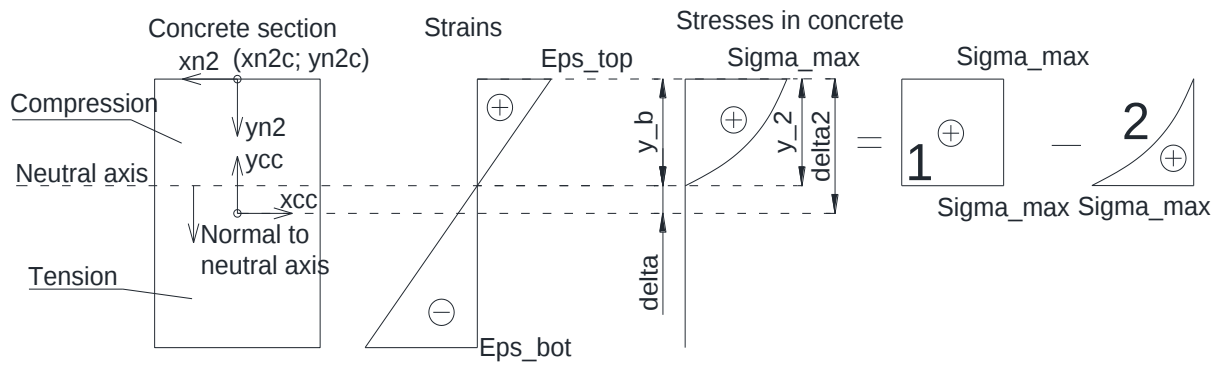
$$y_2 = \text{delta2} - \text{delta} = 0.257143$$

Axial force in compressed part of concrete gross section

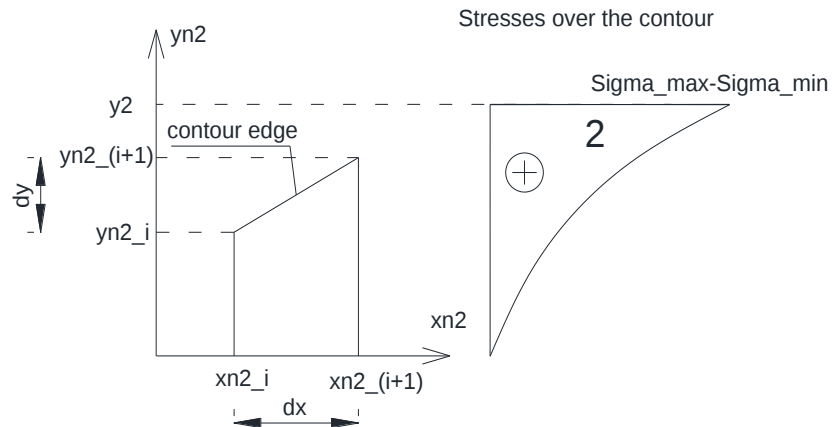
$$N_{c1} = A_c \cdot \text{Sigma_max} = 1.875811\text{e6}$$

To calculate N_{c2} we need to calculate an integral of stresses over contour for which stresses are changing according to the shape 2 on picture below. To do this we define a new coordinate system x_{n2} , y_{n2} (see picture below), where x_{n2} is an axis parallel to neutral axis, in distance delta2 from the gravity center of contour of the section, and it crosses axis y_{n2} in point $(x_{n2c}; y_{n2c})$ given in central axis coordinate system.

$$(x_{n2c}; y_{n2c}) = (0; \text{delta2})$$



Now we find coordinates of contour under stress solid 2 in (x_{n2}, y_{n2}) coordinates system and calculate an integral of stresses over this contour.



i	Coordinates in (x,y) system		Coordinates in central axes system		Coordinates in (X _{n2} , y _{n2}) system		dx	dy	A	B	N _{c2}	Static moment of stress solid		
	x	y	x _{cc}	y _{cc}	x _{n2}	y _{n2}						MS _{y_{n2},z_{n2}}	MS _{x_{n2},z_{n2}}	
1	0,30	0,60	0,15	0,30	-0,15	0,00	0,00	0,26	0,00	0,00	0,000	0,000	0,000	
2	0,30	0,34	0,15	0,04	-0,15	0,26	0,30	0,00	0,00	0,26	0,898	0,000	0,156	
3	0,00	0,34	-0,15	0,04	0,15	0,26	0,00	-0,26	0,00	0,00	0,000	0,000	0,000	
4	0,00	0,60	-0,15	0,30	0,15	0,00	-0,30	0,00	0,00	0,00	0,000	0,000	0,000	
5	0,30	0,60	0,15	0,30	-0,15	0,00	0,00	0,00	0,00	0,00	0,000	0,000	0,000	
	N _{c2} = sum(N _{c2} i) =											0,898		
	C=								0,34 3					
	D=								0,25 7					
gravity center of stress solid 2 in (x _{n2} ; y _{n2}) coordinates system												x _{c2} '	0,0000	
												y _{c2} '	0,1736	
gravity center of stress solid 2 in central axes (x _{cc} ; y _{cc}) coordinates system												x _{c2}	0,0000	
												y _{c2}	0,1264	

Where:

$$x_{cc} = x - x_{Cc}$$

$$y_{cc} = y - y_{Cc}$$

$$x_{n2} = \cos(\text{Angle} - \pi / 2) \cdot (x_{cc} - x_{n2c}) + \sin(\text{Angle} - \pi / 2) \cdot (y_{cc} - y_{n2c})$$

$$y_{n2} = -\sin(\text{Angle} - \pi / 2) \cdot (x_{cc} - x_{n2c}) + \cos(\text{Angle} - \pi / 2) \cdot (y_{cc} - y_{n2c})$$

$$dx = x_{(i+1)} - x_i$$

$$dy = y_{(i+1)} - y_i$$

$$A = dy/dx \text{ if } dx \neq 0 \text{ or } A=0 \text{ if } dx=0$$

$$B = -dy/dx \cdot x_{n2_{(i+1)}} + y_{n2_{(i+1)}} \text{ if } dx \neq 0 \text{ or } B = 0 \text{ if } dx = 0$$

$$C = y_b \cdot \text{Eps_c1} / \text{Eps_top}$$

$$D = \text{MIN}(\text{Eps_c1}; \text{Eps_top}) \cdot y_b / \text{Eps_top}$$

$$n = 1.2 \text{ (power of stress-strain relation curve)}$$

$$d\text{Sigma} = \text{Sigma_max} - \text{Sigma_min}$$

$$N_{c2_i} = 0 \text{ if } A = 0 \text{ and } B = 0 \text{ or}$$

$$N_{c2_i} = f_{cd} \cdot C / (n+1) \cdot (C / (n+2) / A \cdot ((1 - (-y_{n2_{(i+1)}} + D) / C)^{n+2} - (1 - (-y_{n2_i} + D) / C)^{n+2}) - (x_{n2_{(i+1)}} - x_{n2_i}) \cdot (1 - D / C)^{n+1}) + (d\text{Sigma} - f_{cd}) \cdot (A / 2 \cdot (x_{n2_{(i+1)}}^2 - x_{n2_i}^2) + B \cdot (x_{n2_{(i+1)}} - x_{n2_i})) \text{ if } A \neq 0 \text{ or}$$

$$N_{c2_i} = (f_{cd} \cdot C / (n+1) \cdot ((1 - (-B + D) / C)^{n+1} - (1 - D / C)^{n+1})) \cdot dx + (-f_{cd} + d\text{Sigma}) \cdot B \cdot dx \text{ if } A = 0 \text{ and } B \neq 0$$

$$N_{c2} = \text{sum}(N_{c2_i})$$

$$MS_{yn2,zn2_i} = f_{cd} \cdot C / (n+1) \cdot ((1 - (-B + D) / C)^{n+1} - (1 - D / C)^{n+1}) \cdot 1 / 2 \cdot (x_{n2_{(i+1)}}^2 - x_{n2_i}^2) + (-f_{cd} + d\text{Sigma}) \cdot B / 2 \cdot (x_{n2_{(i+1)}}^2 - x_{n2_i}^2) \text{ if } A = 0 \text{ or}$$

$$MS_{yn2,zn2_i} = f_{cd} \cdot C^2 / ((n+1) \cdot (n+2) \cdot A) \cdot (x_{n2_{(i+1)}} \cdot (1 - (-y_{n2_{(i+1)}} + D) / C)^{n+2} - x_{n2_i} \cdot (1 - (-y_{n2_i} + D) / C)^{n+2} - C / (n+3) / A \cdot ((1 - (-y_{n2_{(i+1)}} + D) / C)^{n+3} - (1 - (-y_{n2_i} + D) / C)^{n+3})) - 0.5 \cdot (x_{n2_{(i+1)}}^2 - x_{n2_i}^2) \cdot f_{cd} \cdot C / (n+1) \cdot (1 - D / C)^{n+1} + (-f_{cd} + d\text{Sigma}) \cdot (A / 3 \cdot (x_{n2_{(i+1)}}^3 - x_{n2_i}^3) + B / 2 \cdot (x_{n2_{(i+1)}}^2 - x_{n2_i}^2)) \text{ if } A \neq 0$$

$$MS_{yn2,zn2} = \text{sum}(MS_{yn2,zn2_i})$$

$$MS_{xn2,zn2_i} = (f_{cd} \cdot C / (n+1) \cdot (B \cdot (1 - (-B + D) / C)^{n+1} - C / (n+2) \cdot ((1 - (-B + D) / C)^{n+2} - (1 - D / C)^{n+2})) + (-f_{cd} + d\text{Sigma}) / 2 \cdot B^2) \cdot (x_{n2_{(i+1)}} - x_{n2_i}) \text{ if } A = 0 \text{ or}$$

$$MS_{xn2,zn2_i} = f_{cd} \cdot C^2 / ((n+1) \cdot (n+2) \cdot A) \cdot (y_{n2_{(i+1)}} \cdot (1 - (-y_{n2_{(i+1)}} + D) / C)^{n+2} - y_{n2_i} \cdot (1 - (-y_{n2_i} + D) / C)^{n+2} - C / (n+3) \cdot (2 \cdot (1 - (-y_{n2_{(i+1)}} + D) / C)^{n+3} - 2 \cdot (1 - (-y_{n2_i} + D) / C)^{n+3}) + A \cdot (1 - D / C)^{n+2} \cdot dx) + (d\text{Sigma} - f_{cd}) \cdot (A^2 / 6 \cdot (x_{n2_{(i+1)}}^3 - x_{n2_i}^3) + A \cdot B / 2 \cdot (x_{n2_{(i+1)}}^2 - x_{n2_i}^2) + B^2 / 2 \cdot dx) \text{ if } A \neq 0$$

$$MS_{xn2,zn2} = \text{sum}(MS_{xn2,zn2_i})$$

$$x_{c2}' = MS_{yn2,zn2} / N_{c2}$$

$$y_{c2}' = MS_{xn2,zn2} / N_{c2}$$

$$x_{c2} = x_{n2c} + \cos(\text{Angle} - \pi / 2) \cdot x_{c2}' - \sin(\text{Angle} - \pi / 2) \cdot y_{c2}'$$

$$y_{c2} = y_{n2c} + \sin(\text{Angle} - \pi / 2) \cdot x_{c2}' + \cos(\text{Angle} - \pi / 2) \cdot y_{c2}'$$

$$N_{c2} = 0.897687e6$$

$$N_{c'} = N_{c1} - N_{c2} = 0.978124e6$$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$$x_{c1} = 0$$

$$y_{c1} = \text{delta} + y_b / 2 = 0.171429$$

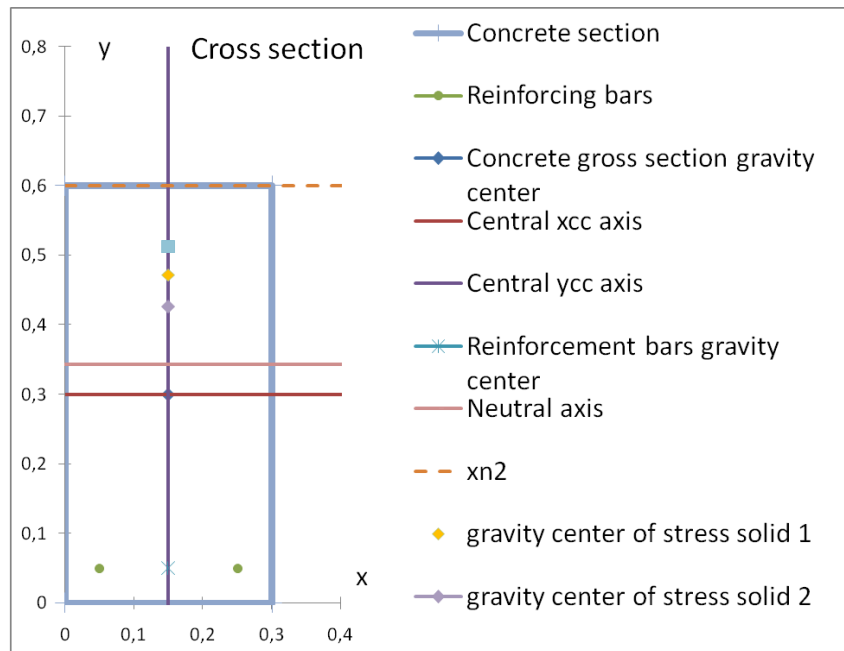
$$x_{c2} = 0$$

$$y_{c2} = 0.126450$$

Center of gravity of stresses in concrete in central axes coordinate system:

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.000000$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.212709$$



Internal forces in concrete:

$$N_c = N_{c'} - N_{sc} = \underline{978.12e3}$$

$$M_{xc} = -N_c \cdot y_{cg} = \underline{-208.06e3}$$

$$M_{yc} = N_c \cdot x_{cg} = \underline{0.00e3}$$

- Results for reduced section:

$$N = N_c + N_s = \underline{428.55e3}$$

$$M_x = M_{xc} + M_{xs} = \underline{-345.45e3}$$

$$M_y = M_{yc} + M_{ys} = \underline{0.00e3}$$

8 Case 7 Calculation of internal forces for given state of strain in the section and inclined branch model of steel

8.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, extreme strains in the section and give internal forces.

8.2 Summary:

This sample gives as results the internal forces for:

- A rectangular concrete shape section with asymmetric reinforcement
- A linear model of concrete and inclined branch model of steel
- A section under unidirectional bending with a part of section under compression
- A state of strain with ultimate compressive strain in concrete

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel.

Based on input data, Component gives as output results:

- Results for reinforcement bars:
 - Axial force (N_s)
 - Moment with respect to x_{cc} axis (M_{xs})
 - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
 - Area of contour under compressive stress (A_c)
 - Center of gravity of stresses in the concrete (x_{cg} , y_{cg})
 - Axial force (N_c)
 - Moment with respect to x_{cc} axis (M_{xc})
 - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
 - Axial force (N)
 - Moment with respect to x_{cc} axis (M_x)
 - Moment with respect to y_{cc} axis (M_y)

For these results, x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

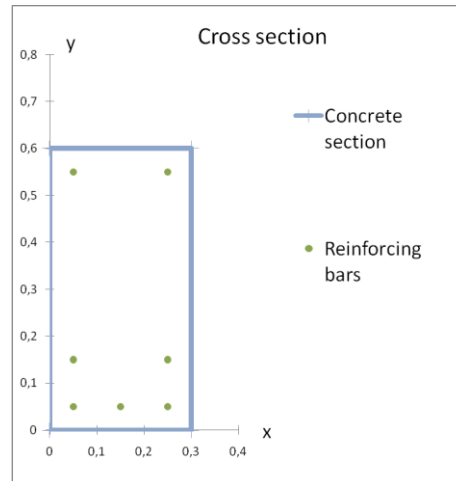
8.3 Data:

- Geometry:
 - Concrete section:

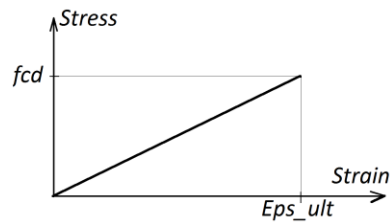
$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.6$
$x = 0.3$	$y = 0.6$
$x = 0.3$	$y = 0.0$
 - Reinforcing bars:

$x = 0.05$	$y = 0.05$	$\phi = 0.032$
$x = 0.15$	$y = 0.05$	$\phi = 0.032$
$x = 0.25$	$y = 0.05$	$\phi = 0.032$

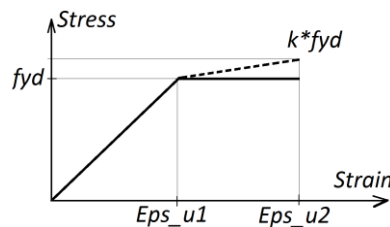
$x = 0.05$	$y = 0.15$	$\phi = 0.020$
$x = 0.25$	$y = 0.15$	$\phi = 0.020$
$x = 0.05$	$y = 0.55$	$\phi = 0.012$
$x = 0.25$	$y = 0.55$	$\phi = 0.012$



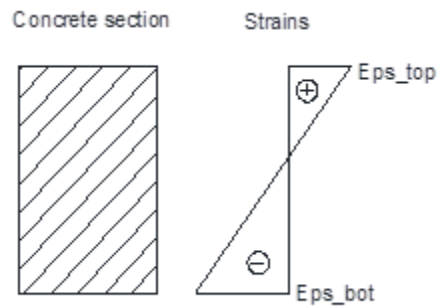
- Concrete parameters:
 - Design strength $f_{cd} = 20e6$
 - Modulus of elasticity $E_c = 30e9$
 - Strain-stress model: linear
 - Strain ultimate limit $Eps_ult = 0.0035$



- Steel parameters:
 - Design strength $f_{yd} = 500e6$
 - Hardening factor $k = 1.05$
 - Modulus of elasticity $E_s = 200e9$
 - Strain ultimate limit $Eps_u2 = 0.075$



- Strains:
 - Top strain $Eps_top = 0.0035$
 - Bottom strain $Eps_bot = -0.0070$
 - Neutral axis angle $Angle = \pi \cdot 3 / 2$



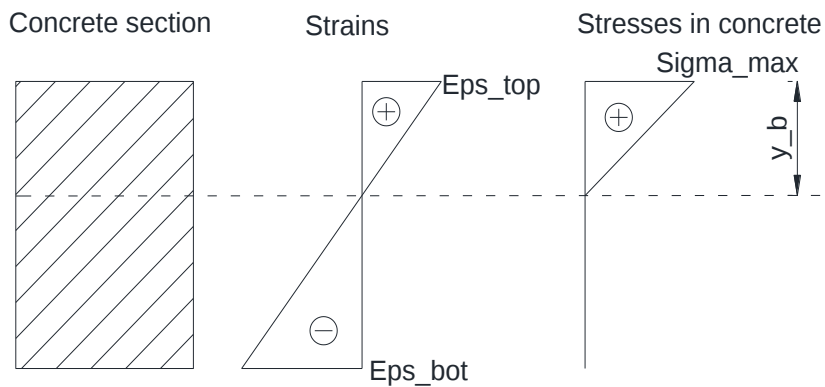
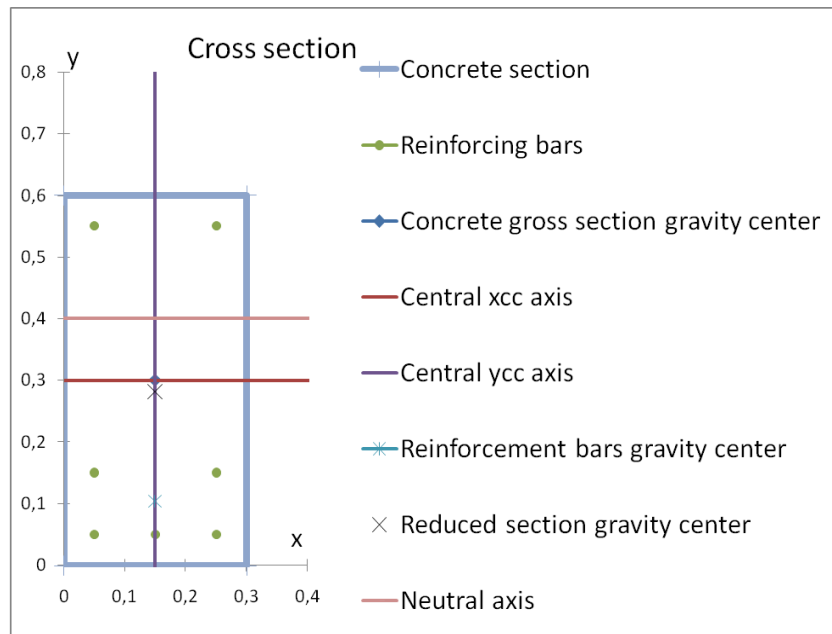
8.4 Search for:

- Results for reinforcement bars: N_s , M_{xs} , M_{ys}
- Results for concrete section: A_c , x_{cg} , y_{cg} , N_c , M_{xc} , M_{yc}
- Results for reduced section: N , M_x , M_y

8.5 Results from Component

- Results for reinforcement bars:
 $N_s = -1410.85e3$
 $M_{xs} = -377.81e3$
 $M_{ys} = 0.00e3$
- Results for concrete section:
 $A_c = 0.06$
 $x_{cg} = 0.00000000$
 $y_{cg} = 0.23323855$
 $N_c = 596.61e3$
 $M_{xc} = -139.15e3$
 $M_{yc} = 0.00e3$
- Results for reduced section:
 $N = -814.24e3$
 $M_x = -516.96e3$
 $M_y = 0.00e3$

8.6 Results (manual calculations):



- Results for reinforcement bars:
Distance between maximum compression corner of the section and neutral axis:

height of a section: $h = 0.6$

$$y_b = -Eps_top \cdot h / (Eps_bot - Eps_top) = 0.2000$$

$$Eps_u1 = f_{yd} / E_s$$

Concrete gross section gravity center:

$$x_{Cc} = 0.15$$

$$y_{Cc} = 0.30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar i:

If $|eps_i| \leq f_{yd} / E_s$ then

$$sigma_i = eps_i / Eps_u1 \cdot f_{yd}$$

else

$$\sigma_i = f_{yd} + (|\epsilon_i| - \epsilon_{u1}) / (\epsilon_{u2} - \epsilon_{u1}) \cdot (k - 1) \cdot f_{yd}$$

i	x	y	ϕ	Distance d from neutral axis	strain $\epsilon = d / y_b \cdot \epsilon_{top}$	stresses $\sigma / 10e6$	force in bar $N = \sigma \cdot \pi \cdot \phi^2 / 4$	Positions in central axes coordinate system x_{cc} y_{cc}		$M_{xs} = -N \cdot y_{cc}$	$M_{ys} = N \cdot x_{cc}$
1	0,05	0,05	0,032	-0,350	-0,00613	-501.25	-403129	-0,10	-0,25	-100782	40313
2	0,15	0,05	0,032	-0,350	-0,00613	-501.25	-403129	0,00	-0,25	-100782	0
3	0,25	0,05	0,032	-0,350	-0,00613	-501.25	-403129	0,10	-0,25	-100782	-40313
4	0,05	0,15	0,02	-0,250	-0,00438	-500.65	-157283	-0,10	-0,15	-23592	15728
5	0,25	0,15	0,02	-0,250	-0,00438	-500.65	-157283	0,10	-0,15	-23592	-15728
6	0,05	0,55	0,012	0,150	0,00263	500.04	56554	-0,10	0,25	-14138	-5655
7	0,25	0,55	0,012	0,150	0,00263	500.04	56554	0,10	0,25	-14138	5655
						Sum of forces in bars:	-1410846			-377808	0

$$N_s = -1410.85e3$$

$$M_{xs} = -377.81e3$$

$$M_{ys} = 0.00e3$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

i	x	y	ϕ	strain ϵ	Positions in central axes coordinate system x_{cc} y_{cc}		Area of concrete with compression stress cut from the section in place of bars	Force N_{sc} in concrete in place of bars $N_{sc} = f_{cd} \cdot \epsilon / \epsilon_{ult} \cdot \text{Area}$	$M_{xsc} = -N_{sc} \cdot y_{cc}$	$M_{ysc} = N_{sc} \cdot x_{cc}$
1	0,05	0,05	0,032	-0,00613	-0,10	-0,25				
2	0,15	0,05	0,032	-0,00613	0,00	-0,25				
3	0,25	0,05	0,032	-0,00613	0,10	-0,25				
4	0,05	0,15	0,02	-0,00438	-0,10	-0,15				
5	0,25	0,15	0,02	-0,00438	0,10	-0,15				
6	0,05	0,55	0,012	0,00263	-0,10	0,25	0,000113097	1696	-424	-170
7	0,25	0,55	0,012	0,00263	0,10	0,25	0,000113097	1696	-424	170
							Sum of forces :	3393	-848	0

Reduction forces in concrete:

$$N_{sc} = 3.39e3$$

$$M_{xsc} = -0.85e3$$

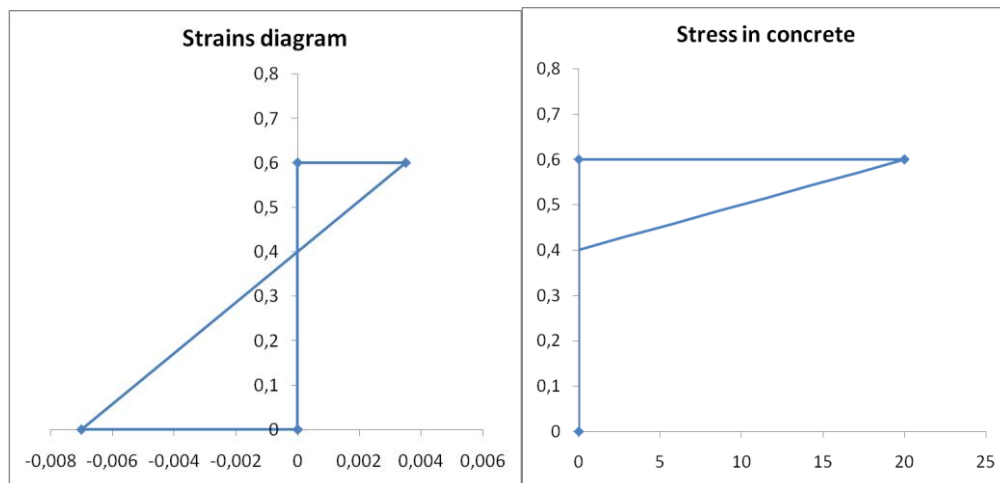
$$M_{ysc} = 0.00e3$$

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

$$x_{scg} = \sum(x_{cc_i} \cdot \text{Area_i}) / \sum(\text{Area_i}) = 0.00$$

$$y_{scg} = \sum(y_{cc_i} \cdot \text{Area_i}) / \sum(\text{Area_i}) = 0.25$$

- Results for concrete section:



Height of compressed part of the section:

$$y_b = 0.20$$

Area of contour under compressive stress:

$$A_c = 0.3 \cdot y_b = 0.0600$$

Axial force in compressed part of concrete gross section

$$\sigma_{\max} = f_{cd} \cdot \epsilon_{\text{top}} / \epsilon_{\text{ult}} = 20.000e6$$

$$N_c = A_c \cdot \sigma_{\max} / 2 = 0.6000e6$$

Center of gravity of stress solid in central axes coordinate system:

$$x'_c = 0$$

$$y'_c = 0.3 - y_b \cdot 2/3 = 0.23658249$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = (x'_c \cdot N_c - x_{scg} \cdot N_{sc}) / (N_c - N_{sc}) = 0.000000$$

$$y_{cg} = (y'_c \cdot N_c - y_{scg} \cdot N_{sc}) / (N_c - N_{sc}) = 0.233239$$

Internal forces in concrete:

$$N_c = N_c - N_{sc} = 596.61e3$$

$$M_{xc} = -N_c \cdot y_{cg} = -139.15e3$$

$$M_{yc} = N_c \cdot x_{cg} = 0.00e3$$

- Results for reduced section:

$$N = N_c + N_s = -814.24e3$$

$$M_x = M_{xc} + M_{xs} = -516.96e3$$

$$M_y = M_{yc} + M_{ys} = 0.00e3$$

9 Case 8: Calculation of internal forces in asymmetric section for given state of strain with horizontal neutral axis

9.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, extreme strains in the section and give internal forces.

9.2 Summary:

This sample gives as results the internal forces for:

- An asymmetric concrete shape section with asymmetric reinforcement
- A rectangular model of concrete and horizontal branch model of steel
- A state of strain with ultimate compressive strain in concrete and horizontal neutral axis

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel.

Based on input data, Component gives as output results:

- Results for reinforcement bars:
 - Axial force (N_s)
 - Moment with respect to x_{cc} axis (M_{xs})
 - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
 - Area of contour under compressive stress (A_c)
 - Center of gravity of stresses in the concrete (x_{cg} , y_{cg})
 - Axial force (N_c)
 - Moment with respect to x_{cc} axis (M_{xc})
 - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
 - Axial force (N)
 - Moment with respect to x_{cc} axis (M_x)
 - Moment with respect to y_{cc} axis (M_y)

For these results, x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

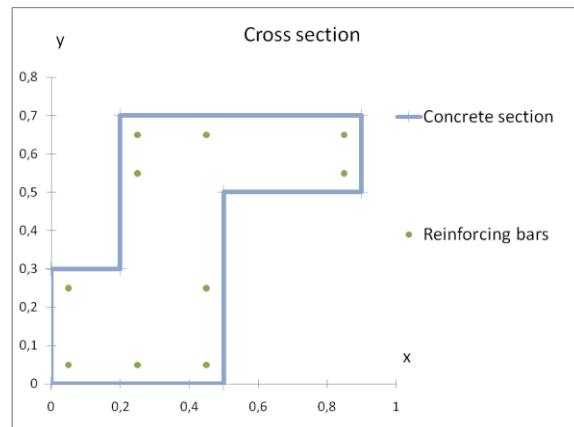
9.3 Data:

- Geometry:
 - Concrete section:

$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.3$
$x = 0.2$	$y = 0.3$
$x = 0.2$	$y = 0.7$
$x = 0.9$	$y = 0.7$
$x = 0.9$	$y = 0.5$
$x = 0.5$	$y = 0.5$
$x = 0.5$	$y = 0.0$

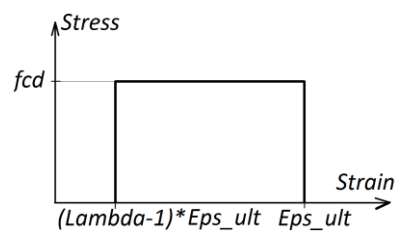
- Reinforcing bars:

$x = 0.05$	$y = 0.05$	$\phi = 0.025$
$x = 0.45$	$y = 0.05$	$\phi = 0.025$
$x = 0.85$	$y = 0.55$	$\phi = 0.012$
$x = 0.85$	$y = 0.65$	$\phi = 0.012$
$x = 0.45$	$y = 0.65$	$\phi = 0.012$
$x = 0.25$	$y = 0.65$	$\phi = 0.012$
$x = 0.25$	$y = 0.05$	$\phi = 0.025$
$x = 0.05$	$y = 0.25$	$\phi = 0.012$
$x = 0.45$	$y = 0.25$	$\phi = 0.012$
$x = 0.25$	$y = 0.55$	$\phi = 0.012$



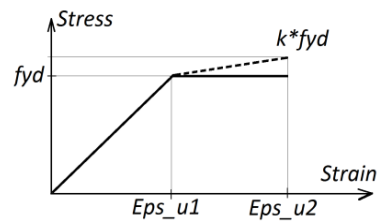
- Concrete parameters:

- Design strength $f_{cd} = 50e6$
- Effective height reduction factor $\Lambda = 0.8$
- Modulus of elasticity $E_c = 37e9$
- Strain-stress model: rectangular
- Strain ultimate limit $Eps_ult = 0.0035$

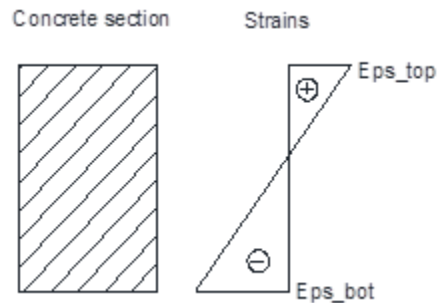


- Steel parameters:

- Design strength $f_{yd} = 500e6$
- Hardening factor $k = 1.0$
- Modulus of elasticity $E_s = 200e9$
- Strain ultimate limit $Eps_u2 = 0.075$



- Strains:
 - Bottom strain Eps_bot = -0.00875
 - Top strain Eps_top = 0.00350
 - Neutral axis angle Angle = $\pi \cdot 3 / 2$



9.4 Search for:

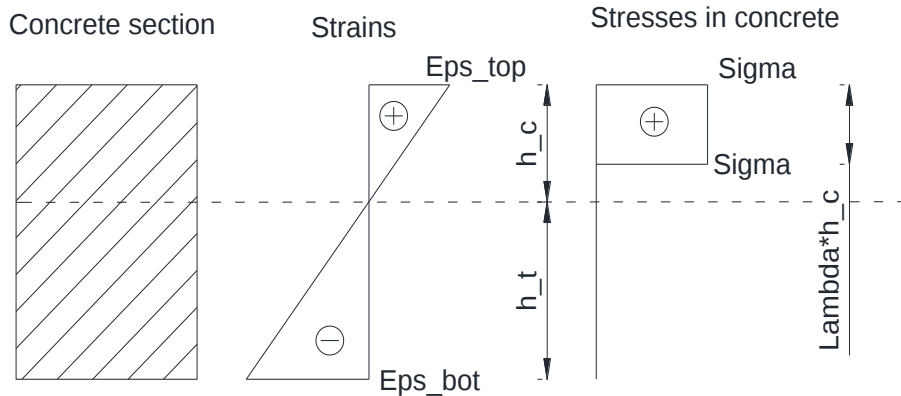
- Results for reinforcement bars: N_s , M_{xs} , M_{ys}
- Results for concrete section: A_c , x_{cg} , y_{cg} , N_c , M_{xc} , M_{yc}
- Results for reduced section: N , M_x , M_y

9.5 Results from Component

- Results for reinforcement bars:
 - $N_s = -640.18e3$
 - $M_{xs} = -305.65e3$
 - $M_{ys} = 144.91e3$
- Results for concrete section:
 - $A_c = 0.1120$
 - $x_{cg} = 0.16295864$
 - $y_{cg} = 0.24719360$
 - $N_c = 5571.73e3$
 - $M_{xc} = -1377.29e3$
 - $M_{yc} = 907.96e3$
- Results for reduced section:
 - $N = 4931.55e3$
 - $M_x = -1682.94e3$
 - $M_y = 1052.87e3$

9.6 Results (manual calculations):

Position of neutral axis



Concrete gross section gravity center calculations:

i	Corners of concrete section		Area_c	Gravity center of contour corresponding to Area_c		Static moments	
	x	y		x_{01}	y_{01}	S_{cy}	S_{cx}
1	0	0	0	0,00	0,15	0	0
2	0	0,3	0,060	0,10	0,15	0,006	0,009
3	0,2	0,3	0	0,20	0,50	0	0
4	0,2	0,7	0,490	0,55	0,35	0,2695	0,1715
5	0,9	0,7	0	0,90	0,60	0	0
6	0,9	0,5	-0,20	0,70	0,25	-0,14	-0,05
7	0,5	0,5	0	0,50	0,25	0	0
8	0,5	0	0	0,25	0,00	0	0
		Total:	0,350			0,1355	0,1305

Where:

edge_i = edge between corners: i and i + 1

Area_c_i = Area between edge_i and axes: $x = x_i$, $x = x_{i+1}$ and $y = 0$

$Area_{c_i} = 1/2 \cdot (y_{i+1} + y_i) \cdot (x_{i+1} - x_i)$

$x_{01_i} = x_i + (x_{i+1} - x_i) \cdot (2/3 \cdot y_{i+1} + 1/3 \cdot y_i) / (y_i + y_{i+1})$ if $y_i + y_{i+1} \neq 0$ or

$x_{01_i} = (x_{i+1} + x_i) / 2$ if $y_i + y_{i+1} = 0$

$y_{01_i} = y_i / 2 + 0.5 \cdot (x_{01_i} - x_i) \cdot (y_{i+1} - y_i) / (x_{i+1} - x_i)$ if $x_{i+1} - x_i \neq 0$ or

$y_{01_i} = (y_{i+1} + y_i) / 2$ if $x_{i+1} - x_i = 0$

$S_{cy_i} = Area_{c_i} \cdot x_{01_i}$

$S_{cx_i} = Area_{c_i} \cdot y_{01_i}$

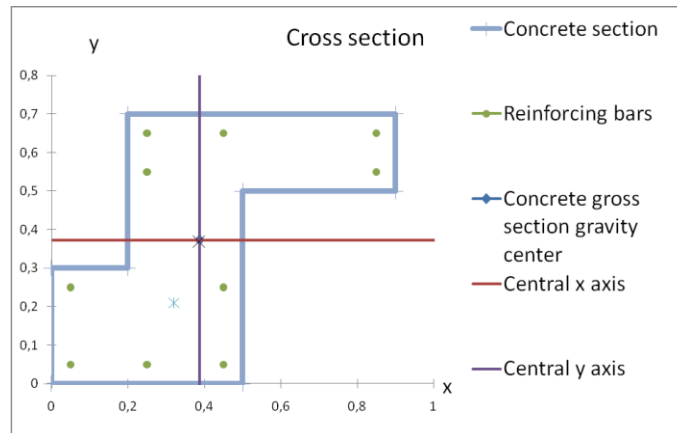
Concrete gross section area:

$$A_c = \sum(Area_{c_i}) = 0,350$$

Concrete gross section gravity center

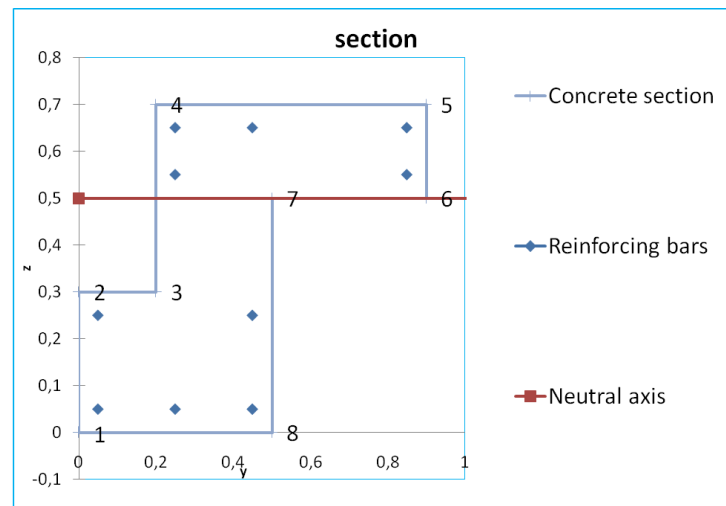
$$x_{Cg} = \sum(S_{cy_i}) / A_c = 0,3871$$

$$y_{Cg} = \sum(S_{cx_i}) / A_c = 0,3729$$



Equation of neutral axis in central axes coordinate system (x_{cc} ; y_{cc})

$y_{cc} = 0.127143$ (e.g. obtained in iteration process)



General equation of neutral axis in central axes coordinate system (x_{cc} ; y_{cc})

$$A \cdot x_{cc} + B \cdot y_{cc} + C = 0$$

$$A = 0$$

$$B = -0.450$$

$$C = 0.057214$$

Distance between corner of concrete section and neutral axis:

For tensioned corner:

$$\text{dist} = (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2 + B^2)^{0.5}$$

For compressed corner:

$$\text{dist} = - (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2 + B^2)^{0.5}$$

i	Corners of concrete section		Corners of concrete section in central axes coordinate system		Corner under compression/tension	Distance between corner and neutral axis dist
	x	y	x_{cc}	y_{cc}		
1	0	0	-0,387	-0,373	tension	0,500
2	0	0,3	-0,387	-0,073	tension	0,200
3	0,2	0,3	-0,187	-0,073	tension	0,200
4	0,2	0,7	-0,187	0,327	compression	-0,200
5	0,9	0,7	0,513	0,327	compression	-0,200
6	0,9	0,5	0,513	0,127	compression	0,000
7	0,5	0,5	0,113	0,127	compression	0,000
8	0,5	0	0,113	-0,373	tension	0,500
Where: $x_{cc} = x - x_{Cc}$ $y_{cc} = y - y_{Cc}$						

Height of compressed part of a section = maximum distance between compressed corner of the section and neutral axis:

$$h_c = |\min(\text{dist}_i)| = 0.2$$

Height of tensioned part of a section = maximum distance between tension corner of the section and neutral axis:

$$h_t = \max(\text{dist}_i) = 0.5$$

Verification of used equation of neutral axis:

$$\text{Eps}_{\text{bot}} = -h_t/h_c \cdot \text{Eps}_{\text{top}} = 0.00875 \text{ (OK)}$$

and

$$\text{neutral axis angle} = \pi + \pi / 2 = 3 / 2 \cdot \pi = \text{Angle (OK)}$$

- Results for reinforcement bars:

$$\text{Eps}_{u1} = f_{yd} / E_s$$

Distance between reinforcing bar and neutral axis:

For tensioned rebar:

$$d = (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2 + B^2)^{0.5}$$

For compressed rebar:

$$d = - (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2 + B^2)^{0.5}$$

Stress in bar i:

If $|\text{eps}_i| \leq f_{yd} / E_s$ then

$$\sigma_i = \text{eps}_i / \text{Eps}_{u1} \cdot f_{yd}$$

else

$$\sigma_i = f_{yd} + (|\text{eps}_i| - \text{Eps}_{u1}) / (\text{Eps}_{u2} - \text{Eps}_{u1}) \cdot (k - 1) \cdot f_{yd}$$

i	x	y	ϕ	Positions in central axes coordinate system		Distance d from neutral axis	strain $\epsilon_s = d/h_c \cdot \epsilon_{s_top}$	stresses $\sigma / 10e6$	force in bar $N = \sigma \cdot \pi \cdot \phi^2 / 4$	$M_{xs} = -N \cdot y_{cc}$	$M_{ys} = N \cdot x_{cc}$
				x_{cc}	y_{cc}						
1	0,05	0,05	0,025	-0,34	-0,32	0,450	-0,00788	-500	-245437	-79241	82747
2	0,45	0,05	0,025	0,06	-0,32	0,450	-0,00788	-500	-245437	-79241	-15427
3	0,85	0,55	0,012	0,46	0,18	-0,050	0,00088	175	19792	-3506	9161
4	0,85	0,65	0,012	0,46	0,28	-0,150	0,00263	500	56549	-15672	26174
5	0,45	0,65	0,012	0,06	0,28	-0,150	0,00263	500	56549	-15672	3554
6	0,25	0,65	0,012	-0,14	0,28	-0,150	0,00263	500	56549	-15672	-7755
7	0,25	0,05	0,025	-0,14	-0,32	0,450	-0,00788	-500	-245437	-79241	33660
8	0,05	0,25	0,012	-0,34	-0,12	0,250	-0,00438	-500	-56549	-6947	19065
9	0,45	0,25	0,012	0,06	-0,12	0,250	-0,00438	-500	-56549	-6947	-3554
10	0,25	0,55	0,012	-0,14	0,18	-0,050	0,00088	175	19792	-3506	-2714
								Sum of forces in bars:	-640178	-305646	144910

$$N_s = -640.18e3$$

$$M_{xs} = -305.65e3$$

$$M_{ys} = 144.91e3$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

i	x	y	ϕ	Positions in central axes coordinate system		Area of concrete with compression stress cut from the section in place of bars	Force N_{sc} in concrete in place of bars $N_{sc} = f_{cd} \cdot \epsilon_{s_top} / \epsilon_{s_ult} \cdot \text{Area}$	$M_{xsc} = -N_{sc} \cdot y_{cc}$	$M_{ysc} = N_{sc} \cdot x_{cc}$
				x_{cc}	y_{cc}				
1	0,05	0,05	0,025	-0,34	-0,32				
2	0,45	0,05	0,025	0,06	-0,32				
3	0,85	0,55	0,012	0,46	0,18	0,000113097	5655	-1002	2617
4	0,85	0,65	0,012	0,46	0,28	0,000113097	5655	-1567	2617
5	0,45	0,65	0,012	0,06	0,28	0,000113097	5655	-1567	355
6	0,25	0,65	0,012	-0,14	0,28	0,000113097	5655	-1567	-776
7	0,25	0,05	0,025	-0,14	-0,32				
8	0,05	0,25	0,012	-0,34	-0,12				
9	0,45	0,25	0,012	0,06	-0,12				
10	0,25	0,55	0,012	-0,14	0,18	0,000113097	5655	-1002	-776
					Total:	0,000565487	28274	-6705	4039

Reduction forces in concrete:

$$N_{sc} = 28.27e3$$

$$M_{xsc} = -6.71e3$$

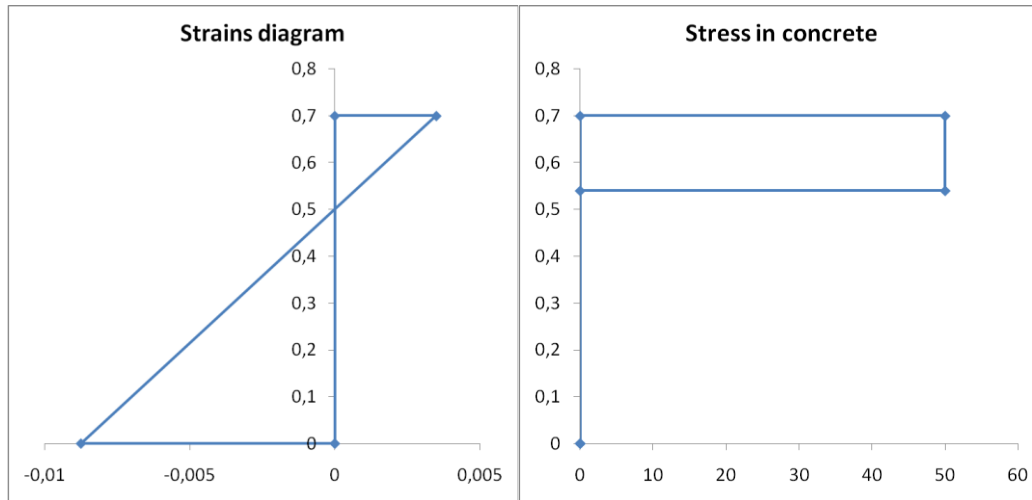
$$M_{y_{sc}} = 4.04e3$$

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

$$x_{scg} = \sum(x_{cc_i} \cdot Area_i) / \sum(Area_i) = 0.1429$$

$$y_{scg} = \sum(y_{cc_i} \cdot Area_i) / \sum(Area_i) = 0.2371$$

- Results for concrete section:



Height of compressed part of the section:

$$\text{Lambda} \cdot h_c = 0.1600$$

Area of contour under compressive stress:

$$A_c = 0.7 \cdot 0.16 = \underline{0.1120}$$

Stress in concrete:

$$\text{Sigma} = \text{Eps_top} / \text{Eps_ult} \cdot f_{cd} = 50e6$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = ((0.2 + 0.7 / 2 - x_{cc}) \cdot A_c \cdot \text{Sigma} - x_{scg} \cdot N_{sc}) / (A_c \cdot \text{Sigma} + N_{sc}) = \underline{0.162959}$$

$$y_{cg} = ((0.7 - 0.2 / 2 - y_{cc}) \cdot A_c \cdot \text{Sigma} - y_{scg} \cdot N_{sc}) / (A_c \cdot \text{Sigma} + N_{sc}) = \underline{0.247194}$$

Internal forces in concrete:

$$N_c = A_c \cdot \text{Sigma} - N_{sc} = \underline{5571.73e3}$$

$$M_{xc} = -N_c \cdot y_{cg} = \underline{-1377.29e3}$$

$$M_{yc} = N_c \cdot x_{cg} = \underline{907.96e3}$$

- Results for reduced section:

$$N = N_c + N_s = \underline{4931.55e3}$$

$$M_x = M_{xc} + M_{xs} = \underline{-1682.94e3}$$

$$M_y = M_{yc} + M_{ys} = \underline{1052.87e3}$$

10 Case 9: Calculation of internal forces in symmetric section for given state of strain with inclined neutral axis

10.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, extreme strains in the section and give internal forces.

10.2 Summary:

This sample gives as results the internal forces for:

- A symmetric concrete shape section with symmetric reinforcement
- A bilinear model of concrete and horizontal branch model of steel
- A state of strain with compressive strain below the ultimate strain in concrete and inclined neutral axis

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel.

Based on input data, Component gives as output results:

- Results for reinforcement bars:
 - Axial force (N_s)
 - Moment with respect to x_{cc} axis (M_{xs})
 - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
 - Area of contour under compressive stress (A_c)
 - Center of gravity of stresses in the concrete (x_{cg} , y_{cg})
 - Axial force (N_c)
 - Moment with respect to x_{cc} axis (M_{xc})
 - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
 - Axial force (N)
 - Moment with respect to x_{cc} axis (M_x)
 - Moment with respect to y_{cc} axis (M_y)

For these results, x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

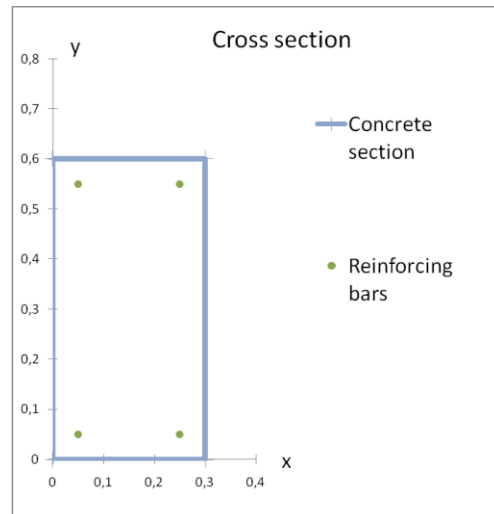
10.3 Data:

- Geometry:
 - Concrete section:

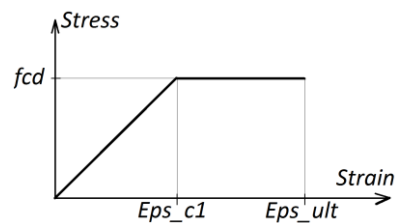
$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.6$
$x = 0.3$	$y = 0.6$
$x = 0.3$	$y = 0.0$
 - Reinforcing bars:

$x = 0.05$	$y = 0.05$	$\phi = 0.012$
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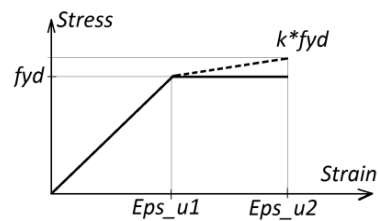
$x = 0.25$	$y = 0.05$	$\phi = 0.012$
$x = 0.25$	$y = 0.55$	$\phi = 0.012$
$x = 0.05$	$y = 0.55$	$\phi = 0.012$



- Concrete parameters:
 - Design strength $f_{cd} = 25e6$
 - Modulus of elasticity $E_c = 28e9$
 - Strain-stress model: bilinear
 - Strain ultimate limit $Eps_ult = 0.0030$
 - Strain relation change over $Eps_c1 = 0.0020$

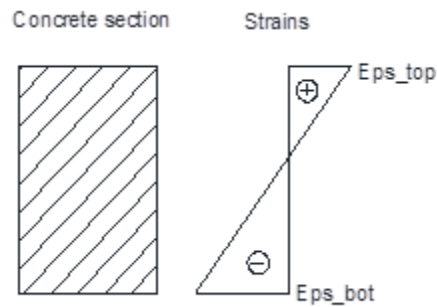


- Steel parameters:
 - Design strength $f_{yd} = 550e6$
 - Hardening factor $k = 1.0$
 - Modulus of elasticity $E_s = 200e9$
 - Strain ultimate limit $Eps_u2 = 0.1$



- Strains:
 - Bottom strain $Eps_bot = -0.00383423$
 - Top strain $Eps_top = 0.0025$

- Neutral axis angle Angle = 0.34906585



10.4 Search for:

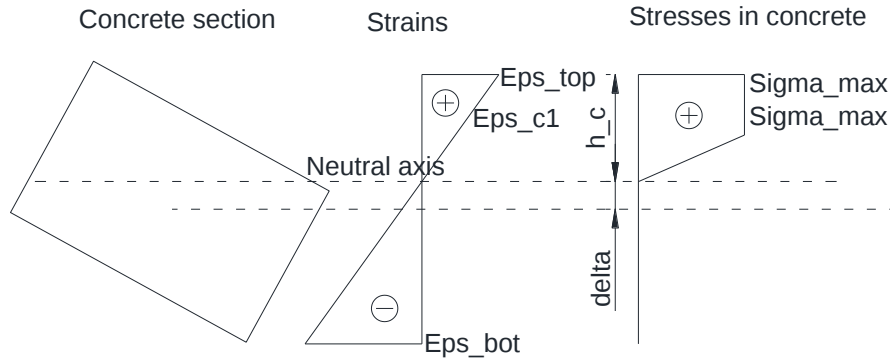
- Results for reinforcement bars: N_s , M_{xs} , M_{ys}
- Results for concrete section: A_c , x_{cg} , y_{cg} , N_c , M_{xc} , M_{yc}
- Results for reduced section: N , M_x , M_y

10.5 Results from Component

- Results for reinforcement bars:
 $N_s = -54.68e3$
 $M_{xs} = 23.73e3$
 $M_{ys} = -10.49e3$
- Results for concrete section:
 $A_c = 0.0575$
 $x_{cg} = -0.09851523$
 $y_{cg} = -0.15819891$
 $N_c = 591.85e3$
 $M_{xc} = 93.63e3$
 $M_{yc} = -58.31e3$
- Results for reduced section:
 $N = 537.17e3$
 $M_x = 117.36e3$
 $M_y = -68.79e3$

10.6 Results (manual calculations):

Position of neutral axis



Concrete gross section gravity center

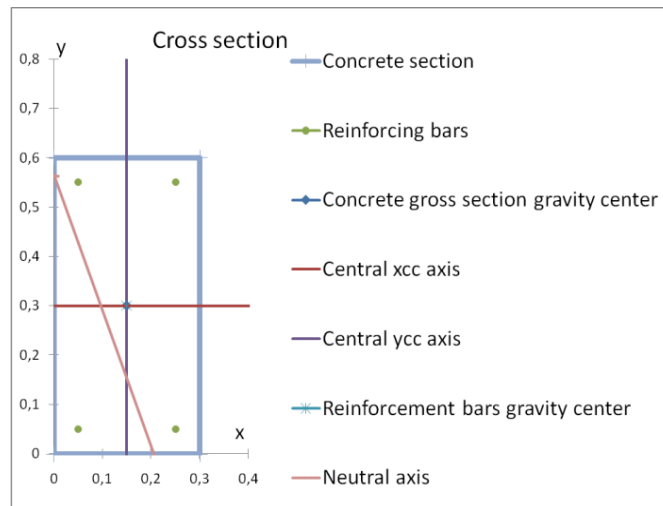
$$x_{Cc} = 0.3 / 2 = 0.15$$

$$y_{Cc} = 0.6 / 2 = 0.30$$

Equation of neutral axis in central axes coordinate system (x_{cc} ; y_{cc})

$$\text{Tangens}(\text{Angle} - \pi / 2) = -2.74748$$

$$y_{cc} = -2.74748 \cdot x_{cc} - 0.15 \text{ (e.g. obtained in iteration process)}$$



General equation of neutral axis in central axes coordinate system (x_{cc} ; y_{cc})

$$A \cdot x_{cc} + B \cdot y_{cc} + C = 0$$

$$A = -0.412$$

$$B = -0.150$$

$$C = -0.0225$$

Distance between corner of concrete section and neutral axis:

For tensioned corner:

$$\text{dist} = (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2 + B^2)^{0.5}$$

For compressed corner:

$$\text{dist} = - (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2 + B^2)^{0.5}$$

i	Corners of concrete section		Corners of concrete section in central axes coordinate system		Corner under compression/tension	Distance between corner and neutral axis dist
	x	y	x_{cc}	y_{cc}		
1	0	0	-0,150	-0,300	compression	-0,19225691
2	0	0,6	-0,150	0,300	tension	0,01295517
3	0,3	0,6	0,150	0,300	tension	0,29486296
4	0,3	0,0	0,150	-0,300	tension	0,08965087
Where: $x_{cc} = x - x_{Cc}$ $y_{cc} = y - y_{Cc}$						

Height of compressed part of a section = maximum distance between compressed corner of the section and neutral axis:

$$h_c = |\min(\text{dist}_i)| = 0.19225691$$

Height of tensioned part of a section = maximum distance between tension corner of the section and neutral axis:

$$h_t = \max(\text{dist}_i) = 0.29486296$$

Verification of used equation of neutral axis:

$$\text{Eps}_{\text{bot}} = -h_t/h_c \cdot \text{Eps}_{\text{top}} = -0.00383423 \text{ (OK)}$$

and

$$\text{neutral axis angle} = \arctan(-2.74748) + \pi/2 = 0.34906585 = \text{Angle (OK)}$$

- Results for reinforcement bars:

$$\text{Eps}_{u1} = f_{yd} / E_s$$

Distance between reinforcing bar and neutral axis:

For tensioned rebar:

$$d = (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2 + B^2)^{0.5}$$

For compressed rebar:

$$d = - (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2 + B^2)^{0.5}$$

Stress in bar i:

If $|\text{eps}_i| \leq f_{yd} / E_s$ then

$$\sigma_i = \text{eps}_i / \text{Eps}_{u1} \cdot f_{yd}$$

else

$$\sigma_i = f_{yd} + (|\text{eps}_i| - \text{Eps}_{u1}) / (\text{Eps}_{u2} - \text{Eps}_{u1}) \cdot (k - 1) \cdot f_{yd}$$

i	x	y	ϕ	Positions in central axes coordinate system		Distance d from neutral axis	strain $\text{eps} = d/h_c \cdot \text{Eps}_{\text{top}}$	stresses $\sigma / 10e6$	force in bar $N = \sigma \cdot \pi \cdot \phi^2 / 4$	$M_{xs} = -N \cdot y_{cc}$	$M_{ys} = N \cdot x_{cc}$
				x_{cc}	y_{cc}						
1	0,05	0,05	0,012	-0,10	-0,25	0,128	0,00167	333	37699	9425	-3770
2	0,25	0,05	0,012	0,10	-0,25	-0,060	-0,00078	-155	-17579	-4395	-1758

3	0,25	0,55	0,012	0,10	0,25	-0,231	-0,00300	-550	-62204	15551	-6220
4	0,05	0,55	0,012	-0,10	0,25	-0,043	-0,00056	-111	-12600	3150	1260
Sum of forces in bars:									-54684	23731	-10488

$$N_s = -54.68e3$$

$$M_{xs} = 23.73e3$$

$$M_{ys} = -10.49e3$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

$$\text{if } \epsilon_s \leq \epsilon_{sc1} \quad N_{sc} = f_{cd} \cdot \epsilon_s / \epsilon_{sc1} \cdot \text{Area}$$

$$\text{if } \epsilon_s > \epsilon_{sc1} \quad N_{sc} = f_{cd} \cdot \text{Area}$$

i	x	y	ϕ	strain eps	Positions in central axes coordinate system		Area of concrete with compression stress cut from the section in place of bars	Force N_{sc} in concrete in place of bars	$M_{xsc} = -$ $N_{sc} \cdot y_{cc}$	$M_{ysc} =$ $N_{sc} \cdot x_{cc}$
					x_{cc}	y_{cc}				
1	0,05	0,05	0,012	0,00167	-0,10	-0,25	0,000113097	2356	589	-236
2	0,25	0,05	0,012	-0,00078	0,10	-0,25				
3	0,25	0,55	0,012	-0,00300	0,10	0,25				
4	0,05	0,55	0,012	-0,00056	-0,10	0,25				
Total:							0,000113097	2356	589	-236

Reduction forces in concrete:

$$N_{sc} = 2.36e3$$

$$M_{xsc} = 0.59e3$$

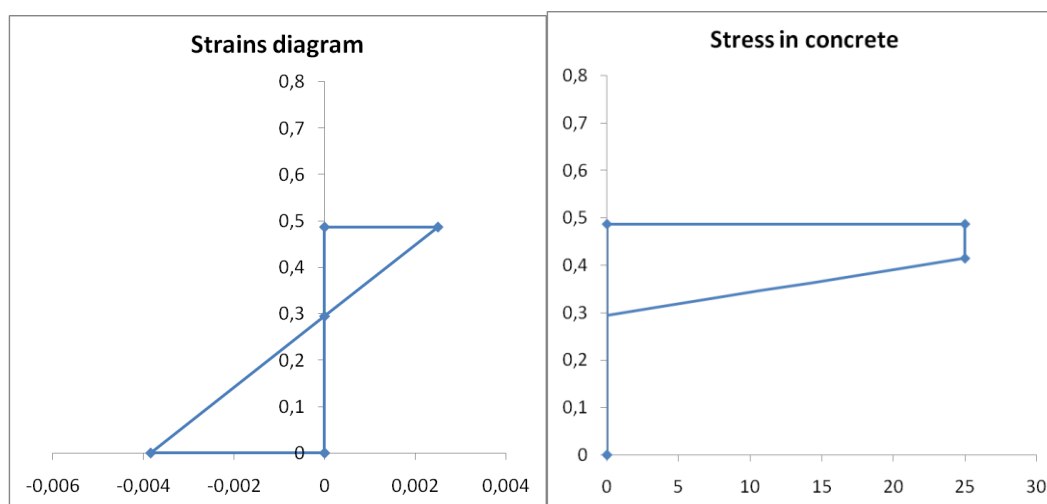
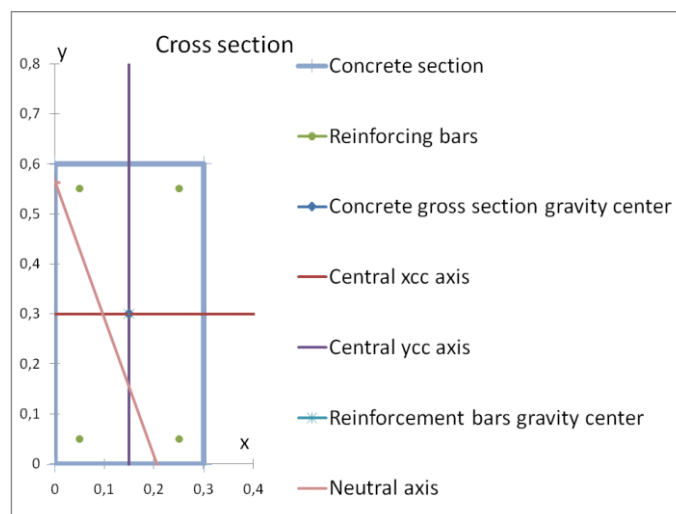
$$M_{ysc} = -0.24e3$$

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

$$x_{scg} = \sum(x_{cc_i} \cdot \text{Area_i}) / \sum(\text{Area_i}) = -0.1000$$

$$y_{scg} = \sum(y_{cc_i} \cdot \text{Area_i}) / \sum(\text{Area_i}) = -0.2500$$

- Results for concrete section:



Calculations of area of contour under compressive stress:

Calculations of points of intersection of neutral axis and the contour of concrete section:

i	Corners of concrete section		Corners of concrete section in central axes coordinate system		Equation of line containing edge_i	Intersection point of line containing edge_i and neutral axis		Is intersection on the contour?
	x	y	x _{cc}	y _{cc}		x _{cc}	y _{cc}	
1	0	0	-0,150	-0,300	x _{cc} = -0,15	-0,15	0,2621	Yes
2	0	0,6	-0,150	0,300	y _{cc} = 0,3	-0,1638	0,3	No
3	0,3	0,6	0,150	0,300	x _{cc} = 0,15	0,15	-0,5621	No
4	0,3	0,0	0,150	-0,300	y _{cc} = -0,3	0,0546	-0,3	Yes

Where:

$$x_{cc} = x - x_{Cc}$$

$$y_{cc} = y - y_{Cc}$$

edge_i = edge between corners: i and i+1

New points created by intersections of lines containing edges and neutral axis and lying on the contour create an edge of contour of compressed part of the section.

i	Contour of compressed part of a section in central axes coordinate system		Contour of compressed part of a section in x y coordinate system		Area_c	Gravity center of contour corresponding to Area_c in (x; y) system		Static moments in (x; y) system	
	x_{cc}	y_{cc}	x	y		x_{01}	y_{01}	S_{cy}	S_{cx}
1	-0,150	-0,300	0,000	0,000	0	0,000	0,281	0,000000	0,000000
2	-0,150	0,262	0,000	0,562	0,0575	0,068	0,187	0,003922	0,010775
3	0,055	-0,300	0,205	0,000	0,0000	0,102	0,000	0,000000	0,000000
4	-0,150	-0,300	0	0	0	0,000	0,281	0,000000	0,000000
				Total:	0,0575			x1=	y1=
								-0,0818	-0,1126

Where:

$$x = x_{cc} + x_{Cc}$$

$$y = y_{cc} + y_{Cc}$$

edge_i = edge between corners: i and i + 1

Area_c_i = Area between edge_i and axes: $x = x_i$, $x = x_{i+1}$ and $y = 0$

$$Area_c_i = 1/2 \cdot (y_{i+1} + y_i) \cdot (x_{i+1} - x_i)$$

$x_{01_i} = x_i + (x_{i+1} - x_i) \cdot (2/3 \cdot y_{i+1} + 1/3 \cdot y_i) / (y_i + y_{i+1})$ if $(y_i + y_{i+1}) \neq 0$ or

$$x_{01_i} = (x_{i+1} + x_i) / 2 \text{ if } (y_i + y_{i+1}) = 0$$

$y_{01_i} = y_i / 2 + 0.5 \cdot (x_{01_i} - x_i) \cdot (y_{i+1} - y_i) / (x_{i+1} - x_i)$ if $x_{i+1} - x_i \neq 0$ or

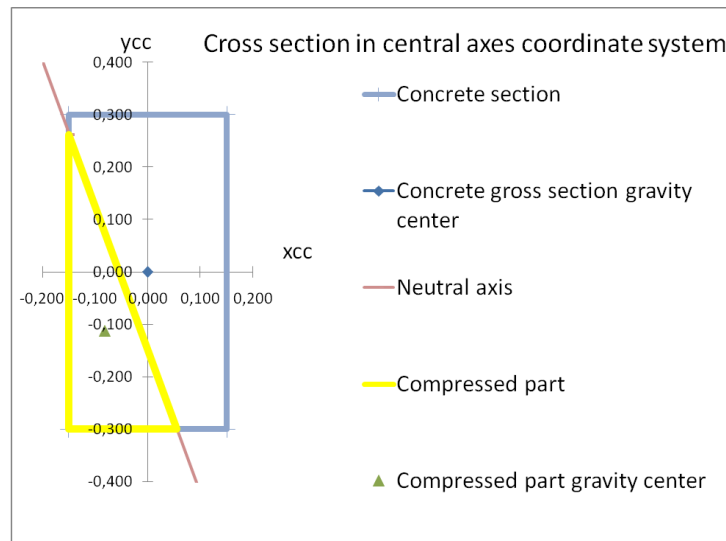
$$y_{01_i} = (y_{i+1} + y_i) / 2 \text{ if } x_{i+1} - x_i = 0$$

$$S_{cy_i} = Area_c_i \cdot x_{01_i}$$

$$S_{cx_i} = Area_c_i \cdot y_{01_i}$$

$$x1 = \text{Sum}(S_{cy}) / \text{sum}(Area_c) - x_{Cc}$$

$$y1 = \text{sum}(S_{cx}) / \text{sum}(Area_c) - y_{Cc}$$



Compressed concrete area:

$$A_c = \text{sum}(\text{Area_c_i}) = \underline{0.0575}$$

Gravity center of compressed concrete contour (gross) in central axes coordinate system

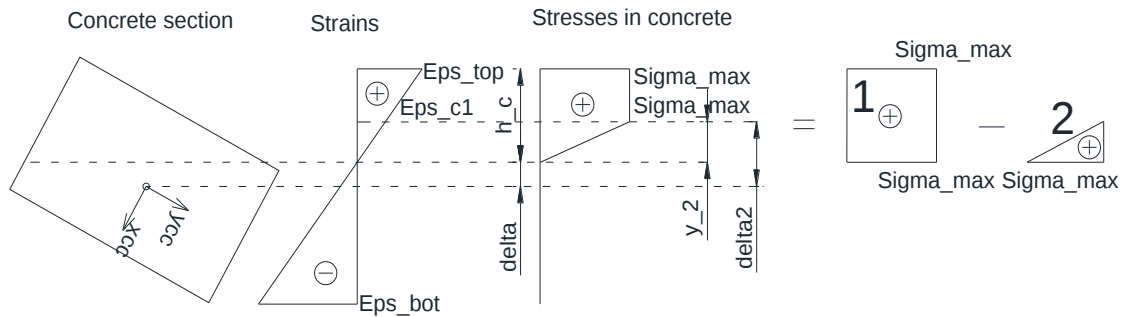
$$x_{c1} = \text{sum}(S_{cy_i}) / A_c - x_{Cc} = -0.0818$$

$$y_{c1} = \text{sum}(S_{cx_i}) / A_c - y_{Cc} = -0.1126$$

Axial force N_c in compressed part of concrete section is calculated by subtraction of volume of a triangular solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of N_c in Introduction).

$$N_{c'} = N_{c1} - N_{c2}$$

$$N_c = N_{c'} - N_{sc}$$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

Moments M_{xc} and M_{yc} caused by stresses acting on entire compressed gross area are calculated following below equations:

$$M_{xc} = -N_c \cdot y_{cg}$$

$$M_{yc} = N_c \cdot x_{cg}$$

$$\text{Sigma_max} = f_{cd} = 25.000\text{e}6$$

Distance between gravity center of contour and neutral axis:

$$\text{delta} = (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2 + B^2)^{0.5}$$

for $x_{cc} = y_{cc} = 0$ and coefficients A, B, C as defined in paragraph: Position of neutral axis, above

$$\text{delta} = 0.0513$$

Distance between the gravity center of contour of the section and:

- the axis with strains equal Eps_c1 if Eps_top $\geq$ Eps_c1 or
- the axis parallel to neutral axis intersecting the most compressed corner of the section if Eps_top < Eps_c1,:

$$\text{delta2} = \text{delta} + \text{MIN}(y_b; \text{Eps_c1} / \text{Eps_top} \cdot y_b) = 0.171352$$

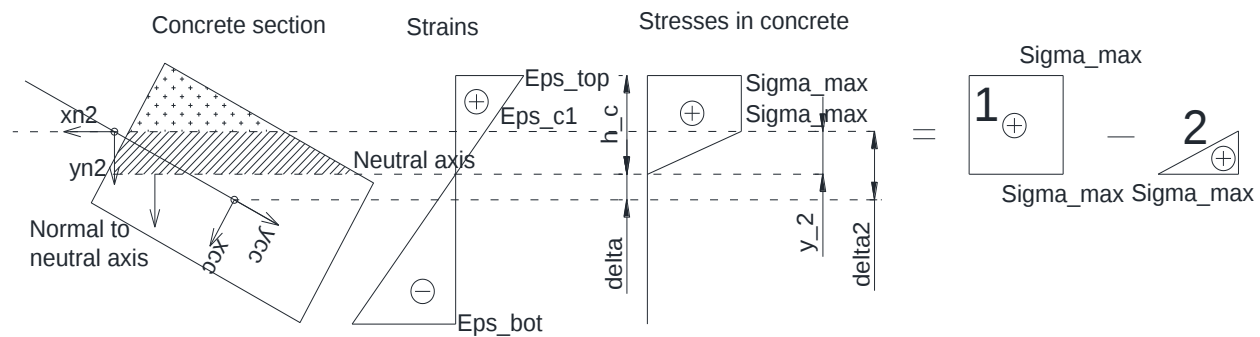
Height of the part of the section where concrete stress is not constant:

$$y_2 = \text{delta2} - \text{delta} = 0.120049$$

Axial force in compressed part of concrete gross section

$$N_{c1} = A_c \cdot \text{Sigma_max} = 1.437595\text{e}6$$

To calculate N_{c2} we need to calculate an integral of stresses over contour for which stresses are changing according to the shape 2 on picture above. To do this we define a new coordinate system x_{n2} , y_{n2} (see picture below), where x_{n2} is an axis parallel to neutral axis, in distance delta2 from the gravity center of contour of the section, and it crosses axis y_{n2} in point $(x_{n2c}; y_{n2c})$ given in central axis coordinate system.



Equation of neutral axis in central axes coordinate system:

$$y_{cc} = -2.74748 \cdot x_{cc} - 0.15$$

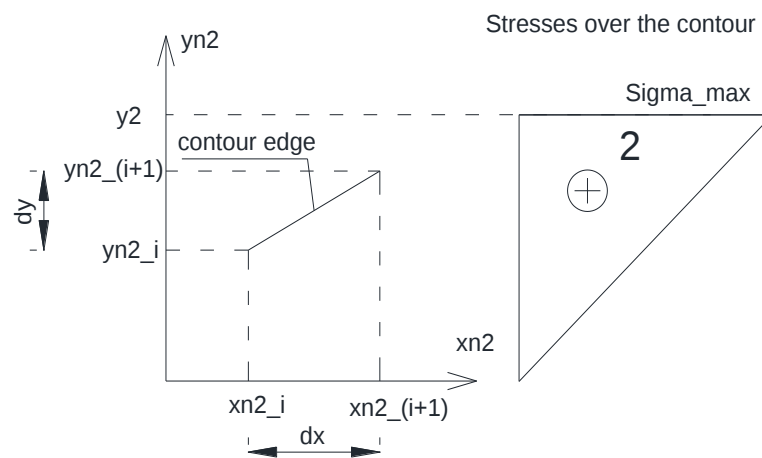
Equation of x_{n2} axis in central axes coordinate system

$$y_{nc} = -2.74748 \cdot x_{nc} - 0.15 - y_2 / \sin(\text{Angle}) = -2.74748 \cdot x_{nc} - 0.5010$$

$$\text{For } x_{n2c} = 0 \text{ } y_{n2c} = -0.5010$$

Now we find coordinates of contour under stress solid 2 in (x_{n2}, y_{n2}) coordinates system and calculate an integral of stresses over this contour.

i	Contour of compressed part of a section		Contour of compressed part of a section in central axes coordinate system		Equation of line containing edge_i, in central axes coordinate system	Intersection point of line containing edge_i and x_{n2} axis		Is intersection on the contour?
	x	y	x_{cc}	y_{cc}		x_{cc}	y_{cc}	
1	0	0	-0,150	-0,300	$x_{cc} = -0,15$	-0,15	-0,0889	Yes
2	0	0,562	-0,15	0,262	$y_{cc} = -2,7475 \cdot x_{cc} - 0,15$	-	-	No
3	0,205	0	0,055	-0,3	$y_{cc} = -0,3$	-0,0732	-0,3	Yes
4	0	0	-0,150	-0,300	$x_{cc} = -0,15$	-0,15	-0,0889	No
Where: $x_{cc} = x - x_{Cc}$ $y_{cc} = y - y_{Cc}$ edge_i = edge between corners: i and i+1								



i	Coordinates of contour below stress solid 2		Coordinates in central axes system		Coordinates in (x _{n2} , y _{n2}) system		dx	dy	A	B	N _{c2}	Static moments of stress solid 2 over the compressed part of concrete section		
	x	y	x _{cc}	y _{cc}	x _{n2}	y _{n2}						MS _{yn2,zn2}	MS _{xn2,zn2}	
1	0,000	0,562	-0,15	0,26	-0,77	0,12	0,60	0,00	0,00	0,12	0,701	-0,329	0,056	
2	0,205	0,000	0,05	-0,30	-0,17	0,12	-0,04	-0,12	2,75	0,59	-0,017	0,003	-0,001	
3	0,077	0,000	-0,07	-0,30	-0,21	0,00	-0,22	0,00	0,00	0,00	0,000	0,000	0,000	
4	0,000	0,211	-0,15	-0,09	-0,44	0,00	-0,33	0,12	-0,36	-0,16	-0,129	0,088	-0,008	
	N _{c2} = sum(N _{c2} i) =											0,555		
	C=									0,15 4				
	D=									0,15 4				
gravity center of stress solid 2 in (x _{n2} ; y _{n2}) coordinates system												x _{c2} '	-0,4279	
												y _{c2} '	0,0853	
gravity center of stress solid 2 in central axes (x _{cc} ; y _{cc}) coordinates system												x _{c2}	-0,0662	
												y _{c2}	-0,0698	

Where:

$$\begin{aligned}
 x_{n2} &= \cos(\text{Angle} - \pi / 2) \cdot (x_{cc} - x_{n2c}) + \sin(\text{Angle} - \pi / 2) \cdot (y_{cc} - y_{n2c}) \\
 y_{n2} &= -\sin(\text{Angle} - \pi / 2) \cdot (x_{cc} - x_{n2c}) + \cos(\text{Angle} - \pi / 2) \cdot (y_{cc} - y_{n2c}) \\
 dx &= x_{n2_}(i + 1) - x_{n2_}i \\
 dy &= y_{n2_}(i + 1) - y_{n2_}i \\
 A &= dy/dx \text{ if } dx \neq 0 \text{ or } A=0 \text{ if } dx=0 \\
 B &= -dy/dx \cdot x_{n2_}(i + 1) + y_{n2_}(i + 1) \text{ if } dx \neq 0 \text{ or } B = 0 \text{ if } dx = 0 \\
 C &= y_b \cdot \text{Eps_c1} / \text{Eps_top} \\
 D &= \text{MIN}(\text{Eps_c1}; \text{Eps_top}) \cdot y_b / \text{Eps_top} \\
 n &= 1.2 \text{ (power of stress-strain relation curve)} \\
 d\text{Sigma} &= \text{Sigma_max} - \text{Sigma_min} \\
 N_{c2_}i &= 0 \text{ if } A = 0 \text{ and } B = 0 \text{ or} \\
 &N_{c2_}i = f_{cd} \cdot C / (n + 1) \cdot (C / (n + 2) / A \cdot ((1 - (-y_{n2_}(i + 1) + D) / C)^{n+2} - (1 - (-y_{n2_}i + D) / C)^{n+2}) - \\
 &\quad (x_{n2_}(i + 1) - x_{n2_}i) \cdot (1 - D / C)^{n+1} + (d\text{Sigma} - f_{cd}) \cdot (A / 2 \cdot (x_{n2_}(i + 1)^2 - x_{n2_}i^2) + B \cdot (x_{n2_}(i + 1) \\
 &\quad - x_{n2_}i)) \text{ if } A \neq 0 \text{ or} \\
 &N_{c2_}i = (f_{cd} \cdot C / (n + 1) \cdot ((1 - (-B + D) / C)^{n+1} - (1 - D / C)^{n+1})) \cdot dx + (-f_{cd} + d\text{Sigma}) \cdot B \cdot \\
 &\quad dx \text{ if } A = 0 \text{ and } B \neq 0 \\
 N_{c2} &= \text{sum}(N_{c2_}i) \\
 MS_{yn2,zn2_}i &= f_{cd} \cdot C / (n + 1) \cdot ((1 - (-B + D) / C)^{n+1} - (1 - D / C)^{n+1}) \cdot 1 / 2 \cdot (x_{n2_}(i + 1)^2 - x_{n2_}i^2) + (-f_{cd} + \\
 d\text{Sigma}) \cdot B / 2 \cdot (x_{n2_}(i + 1)^2 - x_{n2_}i^2) \text{ if } A = 0 \text{ or} \\
 MS_{yn2,zn2_}i &= f_{cd} \cdot C^2 / ((n + 1) \cdot (n + 2) \cdot A) \cdot (x_{n2_}(i + 1) \cdot (1 - (-y_{n2_}(i + 1) + D) / C)^{n+2} - x_{n2_}i \cdot (1 - \\
 (-y_{n2_}i + D) / C)^{n+2} - C / (n + 3) / A \cdot ((1 - (-y_{n2_}(i + 1) + D) / C)^{n+3} - (1 - (-y_{n2_}i + D) / C)^{n+3})) - 0.5 \cdot \\
 (x_{n2_}(i + 1)^2 - x_{n2_}i^2) \cdot f_{cd} \cdot C / (n + 1) \cdot (1 - D / C)^{n+1} + (-f_{cd} + d\text{Sigma}) \cdot (A / 3 \cdot (x_{n2_}(i + 1)^3 - \\
 x_{n2_}i^3) + B / 2 \cdot (x_{n2_}(i + 1)^2 - x_{n2_}i^2)) \text{ if } A \neq 0 \\
 MS_{yn2,zn2} &= \text{sum}(MS_{yn2,zn2_}i) \\
 MS_{xn2,zn2_}i &= (f_{cd} \cdot C / (n + 1) \cdot (B \cdot (1 - (-B + D) / C)^{n+1} - C / (n + 2) \cdot ((1 - (-B + D) / C)^{n+2} - (1 - D / C)^{n+2})) \\
 + (-f_{cd} + d\text{Sigma}) / 2 \cdot B^2) \cdot (x_{n2_}(i + 1) - x_{n2_}i) \text{ if } A = 0 \text{ or} \\
 MS_{xn2,zn2_}i &= f_{cd} \cdot C^2 / ((n + 1) \cdot (n + 2) \cdot A) \cdot (y_{n2_}(i + 1) \cdot (1 - (-y_{n2_}(i + 1) + D) / C)^{n+2} - y_{n2_}i \cdot (1 - \\
 (-y_{n2_}i + D) / C)^{n+2} - C / (n + 3) \cdot (2 \cdot (1 - (-y_{n2_}(i + 1) + D) / C)^{n+3} - 2 \cdot (1 - (-y_{n2_}i + D) / C)^{n+3})) + A \\
 \cdot (1 - D / C)^{n+2} \cdot dx + (d\text{Sigma} - f_{cd}) \cdot (A^2 / 6 \cdot (x_{n2_}(i + 1)^3 - x_{n2_}i^3) + A \cdot B / 2 \cdot (x_{n2_}(i + 1)^2 - \\
 x_{n2_}i^2) + B^2 / 2 \cdot dx) \text{ if } A \neq 0 \\
 MS_{xn2,zn2} &= \text{sum}(MS_{xn2,zn2_}i) \\
 x_{c2}' &= MS_{yn2,zn2} / N_{c2} \\
 y_{c2}' &= MS_{xn2,zn2} / N_{c2} \\
 x_{c2} &= x_{n2c} + \cos(\text{Angle} - \pi / 2) \cdot x_{c2}' - \sin(\text{Angle} - \pi / 2) \cdot y_{c2}' \\
 y_{c2} &= y_{n2c} + \sin(\text{Angle} - \pi / 2) \cdot x_{c2}' + \cos(\text{Angle} - \pi / 2) \cdot y_{c2}'
 \end{aligned}$$

$$N_{c2} = 0.554816e6$$

$$N_{c'} = N_{c1} - N_{c2} = 0.882778e6$$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$$x_{c1} = -0.0818$$

$$y_{c1} = -0.1126$$

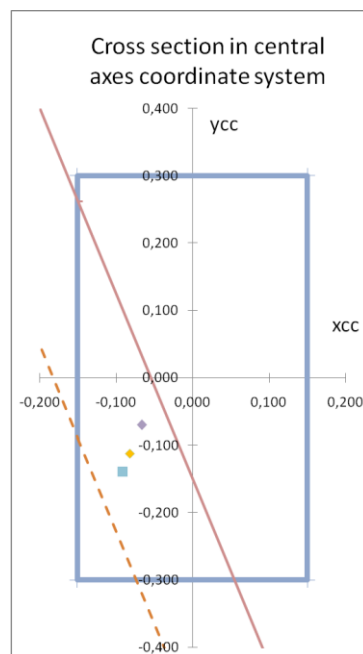
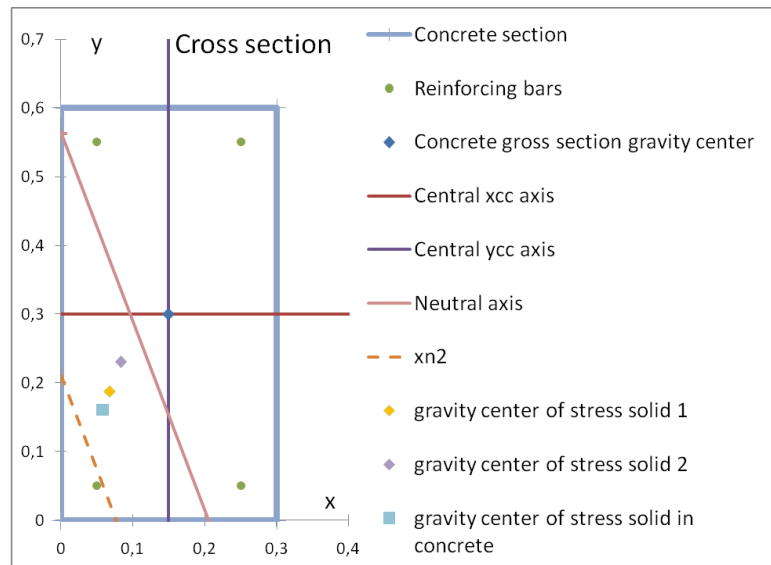
$$x_{c2} = -0.0662$$

$$y_{c2} = -0.0698$$

Center of gravity of stresses in concrete in central axes coordinate system:

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.000000$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.212709$$



Internal forces in concrete:

$$N_c = N_c' - N_{sc} = 880.42e3$$

$$M_{xc} = -N_c \cdot y_{cg} = 122.62e3$$

$$M_{yc} = N_c \cdot x_{cg} = -80.64e3$$

- Results for reduced section:

$$N = N_c + N_s = 825.74e3$$

$$M_x = M_{xc} + M_{xs} = 146.35e3$$

$$M_y = M_{yc} + M_{ys} = -91.12e3$$

11 Case 10: Calculation of capacity state in symmetric section for bending moment M_x

11.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, loads applied to the section ($0, M_x, 0$) and give state of strain and stress at the failure.

11.2 Summary:

This sample gives as results the internal forces for:

- A symmetric concrete section with reinforcement bars
- A rectangular model of concrete stress-strain relation (Case 10a)
- A parabolic-rectangular model of concrete stress-strain relation (Case10b)
- A horizontal branch model of steel
- A simple bending in x direction

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel
- The loads applied to the section

Based on input data, Component gives as output results:

- Results for reinforcement bars:
 - Axial force (N_s)
 - Moment with respect to x_{cc} axis (M_{xs})
 - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
 - Area of contour under compressive stress (A_c)
 - Center of gravity of stresses in the concrete (x_{cg}, y_{cg})
 - Axial force (N_c)
 - Moment with respect to x_{cc} axis (M_{xc})
 - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
 - Axial force at the failure related to acting forces ($N_f = \alpha \cdot N$)
 - Moment at the failure related to acting forces, with respect to x_{cc} axis ($M_{xf} = \alpha \cdot M_x$)
 - Moment at the failure related to acting forces, with respect to y_{cc} axis ($M_{yf} = \alpha \cdot M_y$)

Where alpha is a coefficient called "safety factor".
- Neutral axis position:
 - Distance between the axis and a gravity center of concrete gross section (dist)
 - Angle between horizontal axis and normal to neutral axis directed to the most tensioned fiber (Angle)
- Strains
 - Extreme strains in concrete (Eps_top, Eps_bot)
 - Extreme strains in steel (Eps_stop, Eps_sbot)

For these results, x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

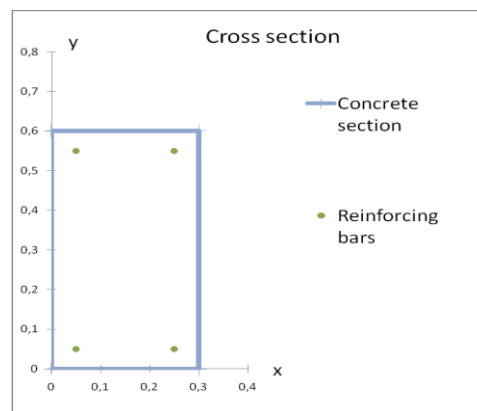
11.3 Case 10a

11.3.1 Data:

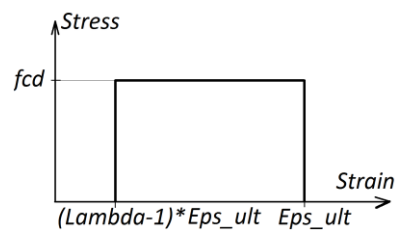
- Geometry:
 - Concrete section:

$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.6$
$x = 0.3$	$y = 0.6$
$x = 0.3$	$y = 0.0$
 - Reinforcing bars:

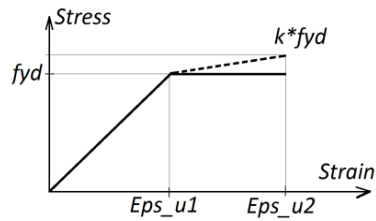
$x = 0.05$	$y = 0.05$	$\phi = 0.020$
$x = 0.25$	$y = 0.05$	$\phi = 0.020$



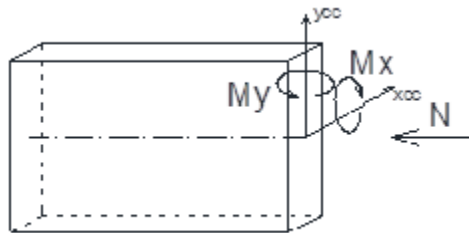
- Concrete parameters:
 - Design strength $f_{cd} = 20e6$
 - Effective height reduction factor $\Lambda = 0.9$
 - Modulus of elasticity $E_c = 30e9$
 - Strain-stress model: rectangular
 - Strain ultimate limit $Eps_ult = 0.0035$



- Steel parameters:
 - Design strength $f_{yd} = 500e6$
 - Hardening factor $k = 1.0$
 - Modulus of elasticity $E_s = 200e9$
 - Strain ultimate limit $Eps_u2 = 0.1$



- Forces:
 - $N = 0.00$
 - $M_x = -50.00$
 - $M_y = 0.00$



11.3.2 Search for:

- Results for reduced section:
 - Forces: N_f , M_{xf} , M_{yf}
 - Neutral axis position: dist, Angle
 - Extreme strains in concrete (Eps_top, Eps_bot)
 - Extreme strains in steel (Eps_stop, Eps_sbot)
- Results for reinforcement bars:
 - Forces: N_s , M_{xs} , M_{ys}
- Results for concrete section:
 - Area of contour under compressive stress: A_c
 - Center of gravity of stresses in the concrete: (x_{cg} , y_{cg})
 - Forces: N_c , M_{xc} , M_{yc}

11.3.3 Results from Component

- Results for reduced section:
 - $N_f = 0.00$
 - $M_{xf} = -164.56$
 - $M_{yf} = 0.00$
 - dist = -0.24182
 - Angle = 4.71239= 270deg
 - Eps_top = 0.003500
 - Eps_bot = -0.032597
 - Eps_stop= -0.029589
 - Eps_sbot= -0.029589
- Results for reinforcement bars:
 - $N_s = -314.16$

$$M_{xs} = -78.54$$

$$M_{ys} = 0.00$$

- Results for concrete section:
 $A_c = 0.0157$
 $(x_{cg}, y_{cg}) = (0.00000000, 0.27382035)$
 $N_c = 314.16$
 $M_{xc} = -86.02$
 $M_{yc} = 0.00$

11.3.4 Results (manual calculations):

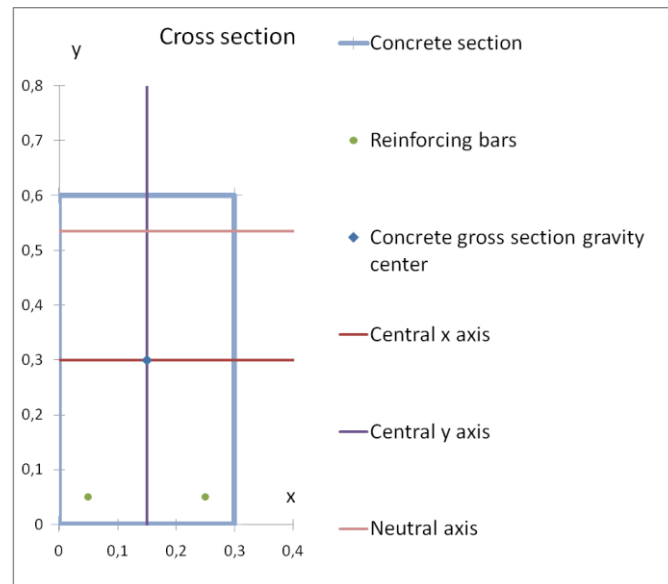
We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.

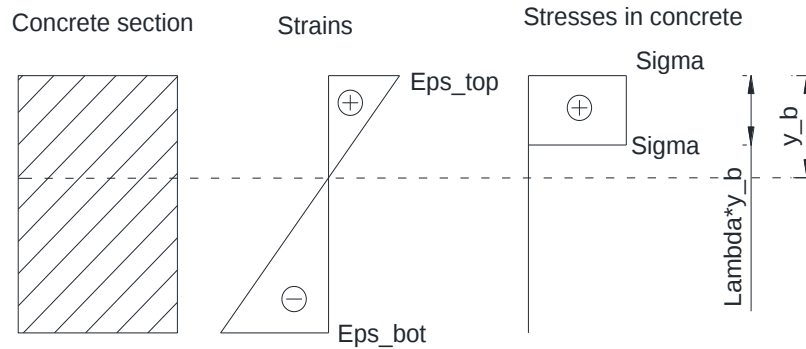
Verification of safety factor:

$$\alpha = M_{xf} / M_x = 3.2912$$

$$N_f = \alpha \cdot N = 0 \text{ (OK)}$$

$$M_{yf} = \alpha \cdot M_y = 0 \text{ (OK)}$$





- Results for reinforcement bars:
Distance between maximum compression corner of the section and neutral axis:
height of a section: $h = 0.6$
 $y_b = -Eps_top \cdot h / (Eps_bot - Eps_top) = 0.05818$
 $Eps_u1 = f_{yd} / E_s$

Concrete gross section gravity center:

$$x_{Cc} = 0.15$$

$$y_{Cc} = 0.30$$

Distance between reinforcing bar and neutral axis:

$$d = y - h + y_b$$

Stress in bar i:

$$\text{If } |eps_i| \leq f_{yd} / E_s \text{ then} \\ \sigma_i = eps_i / Eps_u1 \cdot f_{yd}$$

else

$$\sigma_i = f_{yd} + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) \cdot (k - 1) \cdot f_{yd}$$

i	x	y	ϕ	Positions in central axes coordinate system		Distance d from neutral axis	strain $eps = d/h_c \cdot Eps_top$	stresses $\sigma / 10e6$	force in bar $N = \sigma \cdot \pi \cdot \phi^2 / 4$	$M_{xs} = -N \cdot y_{Cc}$	$M_{ys} = N \cdot x_{Cc}$
				x_{Cc}	y_{Cc}						
1	0,05	0,05	0,02	0,10	-0,25	0,492	-0,02959	-500	-157080	-39270	15708
2	0,25	0,05	0,02	0,10	-0,25	0,492	-0,02959	-500	-157080	-39270	-15708
Sum of forces in bars:									-314159	-78540	0

$$N_s = 314.16e3$$

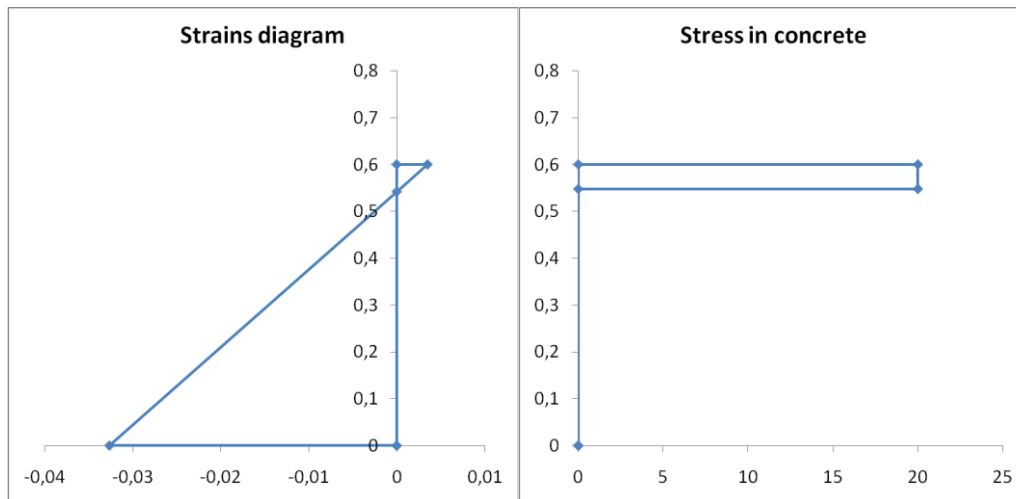
$$M_{xs} = -78.54e3$$

$$M_{ys} = 0.00e3$$

$$Eps_stop = \max(eps) = -0,02959$$

$$Eps_sbot = \min(eps) = -0,02959$$

- Results for concrete section:

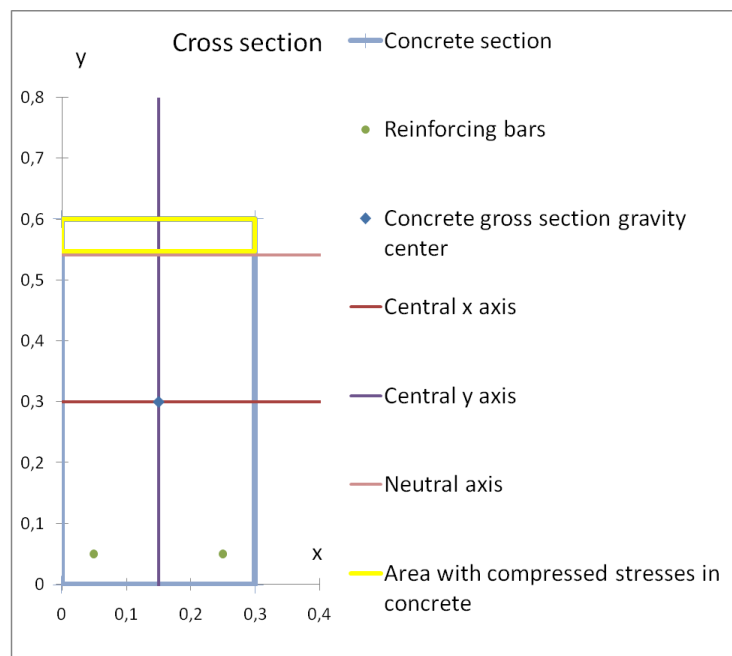


Height of compressed part of the section:

$$\text{Lambda} \cdot y_b = 0.052362$$

Area of contour under compressive stress:

$$A_c = 0.3 \cdot 0.052362 = \underline{0.0157086}$$



Stress in concrete:

$$\text{Sigma} = \text{Eps_top} / \text{Eps_ult} \cdot f_{cd} = 20e6$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = 0.3 / 2 - x_{Cc} = \underline{0.0000}$$

$$y_{cg} = h - \text{Lambda} \cdot y_b / 2 - y_{Cc} = \underline{0.273819}$$

Internal forces in concrete:

$$N_c = A_c \cdot \sigma = 314.17e3$$

$$M_{xc} = -N_c \cdot y_{cg} = -86.03e3$$

$$M_{yc} = N_c \cdot x_{cg} = 0.00e3$$

- Results for reduced section:

$$N_f = N_c + N_s = 0.01e3$$

$$M_{xf} = M_{xc} + M_{xs} = -164.57e3$$

$$M_{yf} = M_{yc} + M_{ys} = 0.00e3$$

Small differences between results from Component and from manual calculations are below 0.01% and are due to precision of iterations.

11.4 Case 10b

11.4.1 Data:

- Geometry:

- Concrete section:

$$x = 0.0 \quad y = 0.0$$

$$x = 0.0 \quad y = 0.6$$

$$x = 0.3 \quad y = 0.6$$

$$x = 0.3 \quad y = 0.0$$

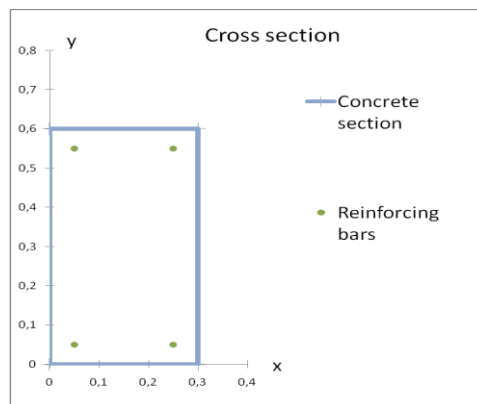
- Reinforcing bars:

$$x = 0.05 \quad y = 0.05 \quad \phi = 0.032$$

$$x = 0.05 \quad y = 0.55 \quad \phi = 0.032$$

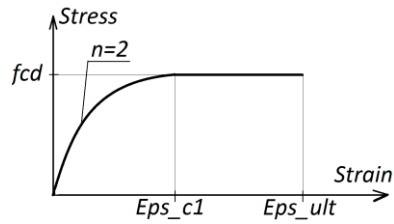
$$x = 0.25 \quad y = 0.55 \quad \phi = 0.032$$

$$x = 0.25 \quad y = 0.05 \quad \phi = 0.032$$

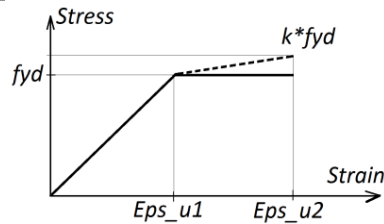


- Concrete parameters:

- Design strength $f_{cd} = 30e6$
- Modulus of elasticity $E_c = 32e9$
- Strain-stress model: parabolic-rectangular
- Strain ultimate limit $Eps_ult = 0.0035$
- Strain relation change over $Eps_c1 = 0.0020$

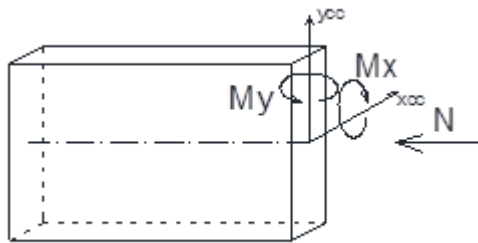


- Steel parameters:
 - Design strength $f_{yd} = 400e6$
 - Hardening factor $k = 1.0$
 - Modulus of elasticity $E_s = 200e9$
 - Strain ultimate limit $Eps_u2 = 0.1$



Forces:

$$\begin{aligned} N &= 0.00 \\ M_x &= -125.00 \\ M_y &= 0.00 \end{aligned}$$



11.4.2 Search for:

- Results for reduced section:
 - Forces: N_f, M_{xf}, M_{yf}
 - Neutral axis position: dist, Angle
 - Extreme strains in concrete (Eps_top, Eps_bot)
 - Extreme strains in steel (Eps_stop, Eps_sbot)
- Results for reinforcement bars:
 - Forces: N_s, M_{xs}, M_{ys}
- Results for concrete section:
 - Area of contour under compressive stress: A_c
 - Center of gravity of stresses in the concrete: (x_{cg}, y_{cg})
 - Forces: N_c, M_{xc}, M_{yc}

11.4.3 Results from Component

- Results for reduced section:
 - $N_f = 0.02$

$M_{xf} = -332.64$
 $M_{yf} = 0.00$
 $dist = -0.237955$
 $Angle = 4.71239 = 270deg$
 $Eps_{top} = 0.003500$
 $Eps_{bot} = -0.030346$
 $Eps_{stop} = 0.000679$
 $Eps_{sbot} = -0.027526$

- Results for reinforcement bars:
 $N_s = -424.81$
 $M_{xs} = -215.50$
 $M_{ys} = 0.00$
- Results for concrete section:
 $A_c = 0.0186$
 $(x_{cg}, y_{cg}) = (0.00000000, 0.27574117)$
 $N_c = 424.83$
 $M_{xc} = -117.14$
 $M_{yc} = 0.00$

11.4.4 Results (manual calculations):

Verification of safety factor:

$\alpha = M_{xf} / M_x = 2.66112$
 $N_f = 0.02 \cong \alpha \cdot N = 0 \text{ (OK)}$
 $M_{yf} = \alpha \cdot M_y = 0 \text{ (OK)}$

We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.

The same neutral axis position, extreme strains and material properties were used as data for Case 5b in this document. Internal forces and strains in the section calculated there are the same as results from Component for this example.

Small differences between results from Component and from manual calculations are below 0.01% and are due to precision of iterations.

12 Case 11: Calculation of capacity state in symmetric section for bending moment M_y

12.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, loads applied to the section $(0, 0, M_y)$ and give state of strain and stress at the failure.

12.2 Summary:

This sample gives as results the internal forces for:

- A symmetric concrete section with reinforcement bars
- A linear model of concrete stress-strain relation (Case 11a)
- A bilinear model of concrete stress-strain relation (Case11b)
- A power-rectangular model of concrete stress-strain relation (Case11c)
- A horizontal branch model of steel
- A simple bending in y direction

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel
- The loads applied to the section

Based on input data, Component gives as output results:

- Results for reinforcement bars:
 - Axial force (N_s)
 - Moment with respect to x_{cc} axis (M_{xs})
 - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
 - Area of contour under compressive stress (A_c)
 - Center of gravity of stresses in the concrete (x_{cg}, y_{cg})
 - Axial force (N_c)
 - Moment with respect to x_{cc} axis (M_{xc})
 - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
 - Axial force at the failure related to acting forces ($N_f = \alpha \cdot N$)
 - Moment at the failure related to acting forces, with respect to x_{cc} axis ($M_{xf} = \alpha \cdot M_x$)
 - Moment at the failure related to acting forces, with respect to y_{cc} axis ($M_{yf} = \alpha \cdot M_y$)

Where alpha is a coefficient called "safety factor".
- Neutral axis position:
 - Distance between the axis and a gravity center of concrete gross section (dist)
 - Angle between horizontal axis and normal to neutral axis directed to the most tensioned fiber (Angle)
- Strains
 - Extreme strains in concrete (Eps_top, Eps_bot)
 - Extreme strains in steel (Eps_stop, Eps_sbot)

For these results, x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

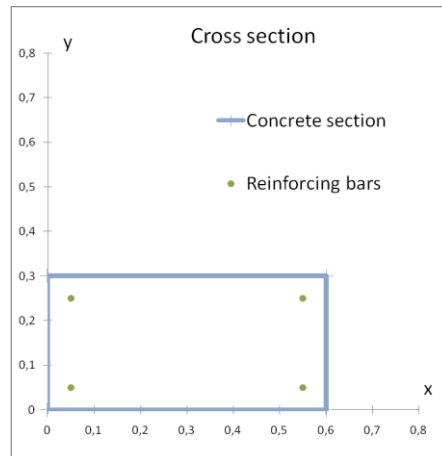
12.3 Case 11a

12.3.1 Data:

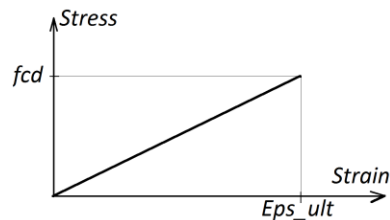
- Geometry:
 - Concrete section:

$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.3$
$x = 0.6$	$y = 0.3$
$x = 0.6$	$y = 0.0$
 - Reinforcing bars:

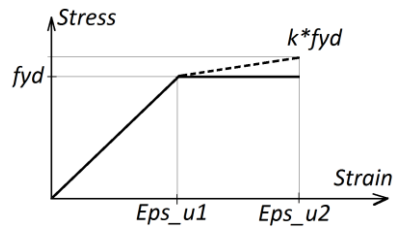
$x = 0.05$	$y = 0.05$	$\phi = 0.032$
$x = 0.05$	$y = 0.25$	$\phi = 0.032$
$x = 0.55$	$y = 0.25$	$\phi = 0.032$
$x = 0.55$	$y = 0.05$	$\phi = 0.032$



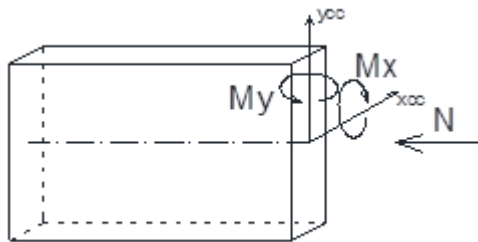
- Concrete parameters:
 - Design strength $f_{cd} = 20e6$
 - Modulus of elasticity $E_c = 30e9$
 - Strain-stress model: linear
 - Strain ultimate limit $Eps_ult = 0.0035$



- Steel parameters:
 - Design strength $f_{yd} = 500e6$
 - Hardening factor $k = 1.0$
 - Modulus of elasticity $E_s = 200e9$
 - Strain ultimate limit $Eps_u2 = 0.075$



- Forces:
 - $N = 0.00 \text{ e3}$
 - $M_x = 0.00 \text{ e3}$
 - $M_y = -600.00 \text{ e3}$



12.3.2 Search for:

- Results for reduced section:
 - Forces: N_f , M_{xf} , M_{yf}
 - Neutral axis position: dist, Angle
 - Extreme strains in concrete (Eps_top, Eps_bot)
 - Extreme strains in steel (Eps_stop, Eps_sbot)
- Results for reinforcement bars:
 - Forces: N_s , M_{xs} , M_{ys}
- Results for concrete section:
 - Area of contour under compressive stress: A_c
 - Center of gravity of stresses in the concrete: (x_{cg}, y_{cg})
 - Forces: N_c , M_{xc} , M_{yc}

12.3.3 Results from Component

- Results for reduced section:
 - $N_f = -0.10 \text{ e3}$
 - $M_{xf} = 0.00 \text{ e3}$
 - $M_{yf} = -407.32 \text{ e3}$
 - dist = -0.204885
 - Angle = 0.00000 = 0.0000deg
 - Eps_top = 0.003500
 - Eps_bot = -0.018572
 - Eps_stop = 0.001661
 - Eps_sbot = -0.016733
- Results for reinforcement bars:
 - $N_s = -270.19 \text{ e3}$

$$M_{xs} = 0.00 \text{ e3}$$

$$M_{ys} = -334.58 \text{ e3}$$

- Results for concrete section:
 $A_c = 0.0285$
 $(x_{cg}, y_{cg}) = (-0.26932850, 0.00000000)$
 $N_c = 270.09 \text{ e3}$
 $M_{xc} = 0.00 \text{ e3}$
 $M_{yc} = -72.74 \text{ e3}$

12.3.4 Results (manual calculations):

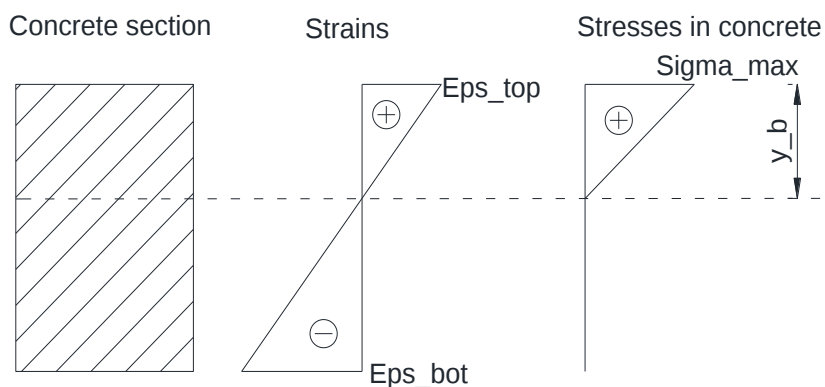
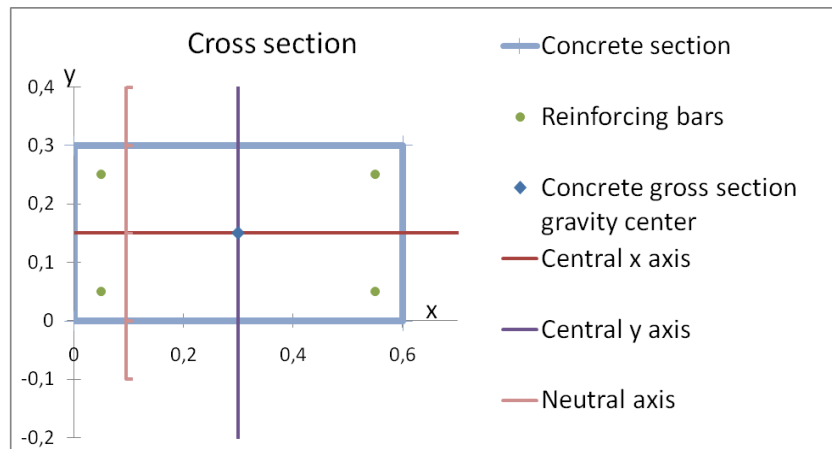
Verification of safety factor:

$$\alpha = M_{yf} / M_y = 0.67887$$

$$N_f = -0.10 \cong \alpha \cdot N = 0 \text{ (OK)}$$

$$M_{xf} = \alpha \cdot M_x = 0 \text{ (OK)}$$

We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.



Results for reinforcement bars:

Distance between maximum compression corner of the section and neutral axis:

width of a section: $b = 0.6$

$$y_b = -Eps_top \cdot b / (Eps_bot - Eps_top) = 0.095144$$

$$Eps_u1 = f_{yd} / E_s$$

Concrete gross section gravity center:

$$x_{Cc} = 0.30$$

$$y_{Cc} = 0.15$$

Distance between reinforcing bar and neutral axis:

$$d = y - b + y_b$$

Stress in bar i:

If $|eps_i| \leq f_{yd} / E_s$ then

$$sigma_i = eps_i / Eps_u1 \cdot f_{yd}$$

else

$$sigma_i = f_{yd} + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) \cdot (k - 1) \cdot f_{yd}$$

i	x	y	ϕ	Distance d from neutral axis	strain $eps = d /$ $y_b \cdot$ Eps_top	stresses $sigma /$ $10e6$	force in bar $N =$ $sigma \cdot \pi$ $\cdot \phi^2 / 4$	Positions in central axes coordinate system x_{Cc} y_{Cc}		$M_{xs} = -N$ $\cdot y_{Cc}$	$M_{ys} = N \cdot$ x_{Cc}
1	0,05	0,05	0,032	-0,045	0,00166	332	267120	-0,25	-0,10	26712	-66780
2	0,55	0,05	0,032	0,455	-0,01673	-500	-402124	0,25	-0,10	-40212	-100531
3	0,55	0,25	0,032	0,455	-0,01673	-500	-402124	0,25	0,10	40212	-100531
4	0,05	0,25	0,032	-0,045	0,00166	332	267120	-0,25	0,10	-26712	-66780
						Sum of forces in bars:	-270008			0	-334622

$$N_s = -270.01e3$$

$$M_{xs} = 0.00e3$$

$$M_{ys} = -334.62e3$$

$$Eps_stop = \max(eps) = 0.00166$$

$$Eps_sbot = \min(eps) = -0.01673$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

i	x	y	ϕ	strain eps	Positions in central axes coordinate system x_{Cc} y_{Cc}		Area of concrete with compression stress cut from the section in place of bars	Force N_{sc} in concrete in place of bars $N_{sc} = f_{cd} \cdot$ $eps / Eps_ult \cdot$ Area	$M_{xsc} = -$ $N_{sc} \cdot y_{Cc}$	$M_{ysc} =$ $N_{sc} \cdot x_{Cc}$
1	0,05	0,05	0,032	0,00166	-0,25	-0,10	0,000804248	7632	763	-1908
2	0,55	0,05	0,032	-0,01673	0,25	-0,10				
3	0,55	0,25	0,032	-0,01673	0,25	0,10				
4	0,05	0,25	0,032	0,00166	-0,25	0,10	0,000804248	7632	-763	-1908
							Sum of forces :	15264	0	-3816

Reduction forces in concrete:

$$N_{sc} = 15.26e3$$

$$M_{xsc} = 0.00e3$$

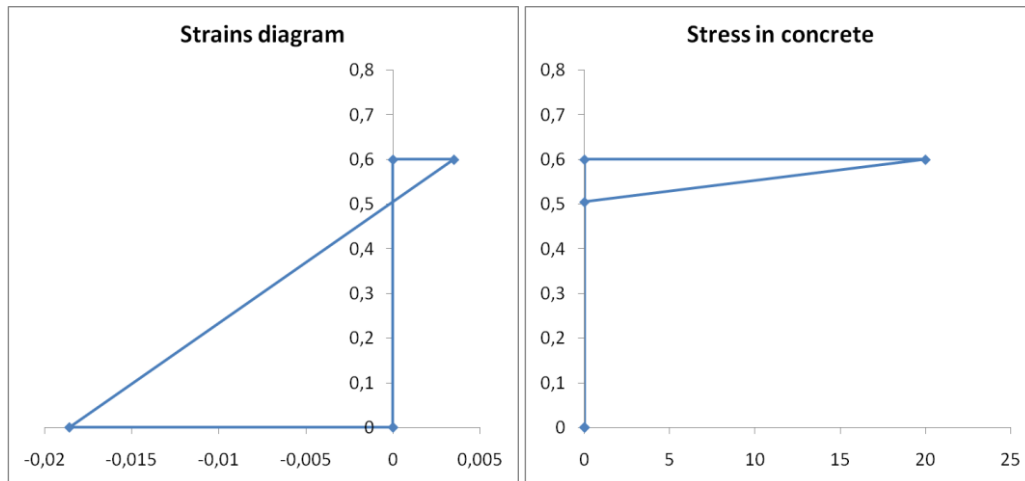
$$M_{ysc} = -3.82e3$$

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

$$x_{scg} = \sum(x_{cc_i} \cdot Area_i) / \sum(Area_i) = -0.25$$

$$y_{scg} = \sum(y_{cc_i} \cdot Area_i) / \sum(Area_i) = 0.00$$

- Results for concrete section:



Height of compressed part of the section:

$$y_b = 0.095144$$

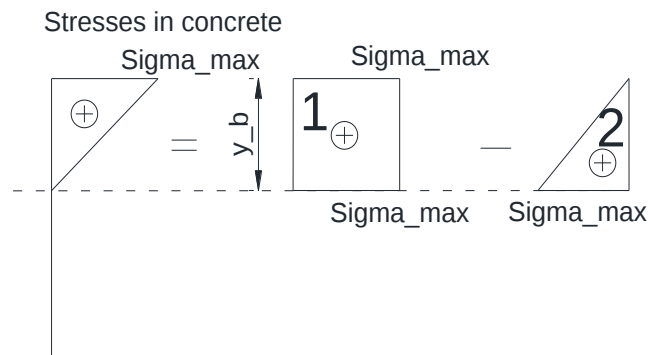
Area of contour under compressive stress:

$$A_c = 0.3 \cdot y_b = 0.0285$$

Axial force N_c in compressed part of concrete section is calculated by subtraction of volume of a triangular solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of N_c in Introduction).

$$N_{c'} = N_{c1} - N_{c2}$$

$$N_c = N_{c'} - N_{sc}$$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of

stresses in solids are used as wages.

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

Moments M_{xc} and M_{yc} caused by stresses acting on entire compressed gross area are calculated following below equations:

$$M_{xc} = - N_c \cdot y_{cg}$$

$$M_{yc} = N_c \cdot x_{cg}$$

$$\text{Sigma_max} = f_{cd} \cdot \text{Eps_top} / \text{Eps_ult} = 20.000\text{e}6$$

Axial force in compressed part of concrete gross section

$$N_{c1} = A_c \cdot \text{Sigma_max} = 0.5709\text{e}6$$

$$N_{c2} = A_c \cdot \text{Sigma_max} / 2 = 0.2854\text{e}6$$

$$N_{c'} = N_{c1} - N_{c2} = 0.285432\text{e}6$$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$$x_{c1} = -0.3 + y_b / 2 = -0.252428$$

$$y_{c1} = 0$$

$$x_{c2} = -0.3 + y_b \cdot 2/3 = -0.236570$$

$$y_{c2} = 0$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = \underline{0.269318}$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = \underline{0.000000}$$

Internal forces in concrete:

$$N_c = N_{c'} - N_{sc} = \underline{270.17\text{e}3}$$

$$M_{xc} = - N_c \cdot y_{cg} = \underline{0.00\text{e}3}$$

$$M_{yc} = N_{c'} \cdot x_{cg} = \underline{-72.76\text{e}3}$$

- Results for reduced section:

$$N_f = N_c + N_s = \underline{0.16\text{e}3}$$

$$M_{xf} = M_{xc} + M_{xs} = \underline{0.00\text{e}3}$$

$$M_{yf} = M_{yc} + M_{ys} = \underline{-407.38\text{e}3}$$

Small differences between results from Component and from manual calculations are below 0.1% and are due to precision of iterations.

12.4 Case 11b

12.4.1 Data:

- Geometry:

- Concrete section:

$$x = 0.0 \quad y = 0.0$$

$$x = 0.0 \quad y = 0.3$$

$$x = 0.6 \quad y = 0.3$$

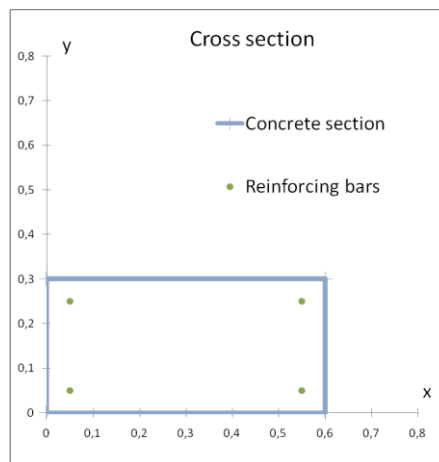
$$x = 0.6 \quad y = 0.0$$

- Reinforcing bars:

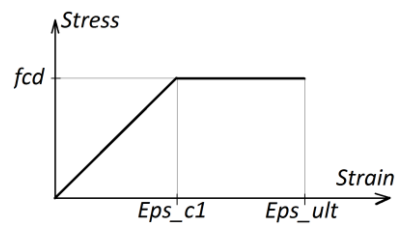
$$x = 0.05 \quad y = 0.05 \quad \phi = 0.032$$

$$x = 0.05 \quad y = 0.25 \quad \phi = 0.032$$

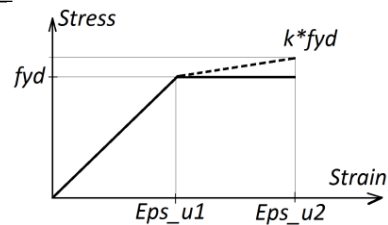
$$\begin{array}{lll} x = 0.55 & y = 0.25 & \phi = 0.032 \\ x = 0.55 & y = 0.05 & \phi = 0.032 \end{array}$$



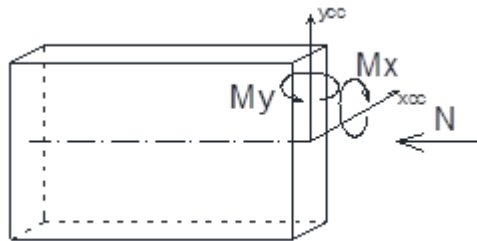
- Concrete parameters:
 - Design strength $f_{cd} = 30e6$
 - Modulus of elasticity $E_c = 32e9$
 - Strain-stress model: bilinear
 - Strain ultimate limit $Eps_ult = 0.0035$
 - Strain relation change over $Eps_c1 = 0.0020$



- Steel parameters:
 - Design strength $f_{yd} = 400e6$
 - Hardening factor $k = 1.0$
 - Modulus of elasticity $E_s = 205e9$
 - Strain ultimate limit $Eps_u2 = 0.1$



- Forces:
 - $N = 0.00 \text{ e3}$
 - $M_x = 0.00 \text{ e3}$
 - $M_y = -10.00 \text{ e3}$



12.4.2 Search for:

- Results for reduced section:
 - Forces: N_f , M_{xf} , M_{yf}
 - Neutral axis position: dist, Angle
 - Extreme strains in concrete (Eps_top, Eps_bot)
 - Extreme strains in steel (Eps_stop, Eps_sbot)
- Results for reinforcement bars:
 - Forces: N_s , M_{xs} , M_{ys}
- Results for concrete section:
 - Area of contour under compressive stress: A_c
 - Center of gravity of stresses in the concrete: (x_{cg}, y_{cg})
 - Forces: N_c , M_{xc} , M_{yc}

12.4.3 Results from Component

- Results for reduced section:
 - $N_f = 0.09 \text{ e3}$
 - $M_{xf} = 0.00 \text{ e3}$
 - $M_{yf} = -332.39 \text{ e3}$
 - dist = -0.236099
 - Angle = 0.000000 = 0.000000 deg
 - Eps_top = 0.003500
 - Eps_bot = -0.029363
 - Eps_stop = 0.000761
 - Eps_sbot = -0.026625
- Results for reinforcement bars:
 - $N_s = -392.34 \text{ e3}$
 - $M_{xs} = 0.00 \text{ e3}$
 - $M_{ys} = -223.61 \text{ e3}$
- Results for concrete section:
 - $A_c = 0.0192$
 - $(x_{cg}, y_{cg}) = (-0.27717502, 0.00000000)$
 - $N_c = 392.42 \text{ e3}$
 - $M_{xc} = 0.00 \text{ e3}$
 - $M_{yc} = -108.77 \text{ e3}$

12.4.4 Results (manual calculations):

Verification of safety factor:

$$\begin{aligned}\alpha &= M_{yfl} / M_y = 33.239 \\ N_f &= 0.09 \cong \alpha \cdot N = 0 \text{ (OK)} \\ M_{xf} &= \alpha \cdot M_x = 0 \text{ (OK)}\end{aligned}$$

We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.

Example Case 4b in this document presents calculations of internal forces for the section and stress-strain relation (both the same as in this example after rotation -90deg), and with the same material parameters and extreme strains. Appropriate internal forces and strains in the section calculated there are the same as a corresponding results from Component for this example.

Small differences between results from Component and from manual calculations are below 0.03% and are due to precision of iterations.

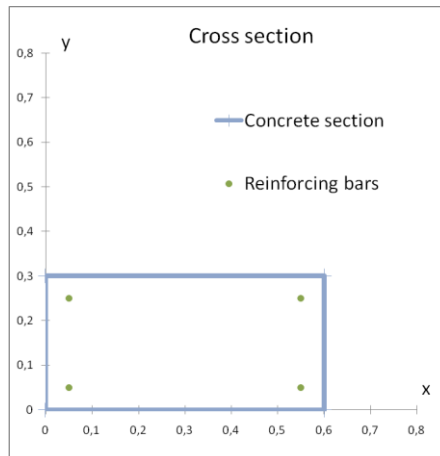
12.5 Case 11c

12.5.1 Data:

- Geometry:
 - Concrete section:

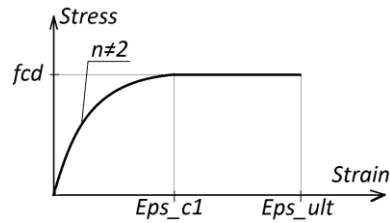
$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.3$
$x = 0.6$	$y = 0.3$
$x = 0.6$	$y = 0.0$
- Reinforcing bars:

$x = 0.05$	$y = 0.05$	$\phi = 0.032$
$x = 0.05$	$y = 0.25$	$\phi = 0.032$
$x = 0.55$	$y = 0.25$	$\phi = 0.032$
$x = 0.55$	$y = 0.05$	$\phi = 0.032$

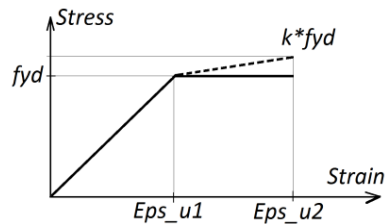


- Concrete parameters:
 - Design strength $f_{cd} = 30e6$
 - Modulus of elasticity $E_c = 32e9$
 - Strain-stress model: power-rectangular
 - Power: $n = 1.8$

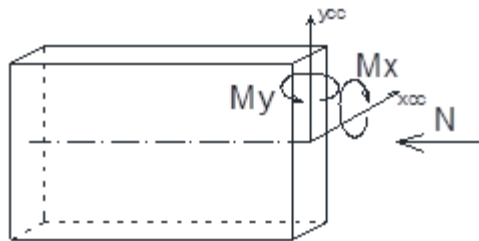
- Strain ultimate limit $Eps_ult = 0.0035$
- Strain relation change over $Eps_c1 = 0.0020$



- Steel parameters:
 - Design strength $f_{yd} = 400e6$
 - Hardening factor $k = 1.0$
 - Modulus of elasticity $E_s = 200e9$
 - Strain ultimate limit $Eps_u2 = 0.1$



- Forces:
 - $N = 0.00\ e3$
 - $M_x = 0.00\ e3$
 - $M_y = -10.00\ e3$



12.5.2 Search for:

- Results for reduced section:
 - Forces: N_f , M_{xf} , M_{yf}
 - Neutral axis position: dist, Angle
 - Extreme strains in concrete (Eps_top , Eps_bot)
 - Extreme strains in steel (Eps_stop , Eps_sbot)
- Results for reinforcement bars:
 - Forces: N_s , M_{xs} , M_{ys}
- Results for concrete section:
 - Area of contour under compressive stress: A_c
 - Center of gravity of stresses in the concrete: (x_{cg} , y_{cg})
 - Forces: N_c , M_{xc} , M_{yc}

12.5.3 Results from Component

- Results for reduced section:
 $N_f = -0.16 \text{ e3}$
 $M_{xf} = 0.00 \text{ e3}$
 $M_{yf} = -332.56 \text{ e3}$
 $\text{dist} = -0.237679$
 $\text{Angle} = 0.00000 = 0.00000 \text{ deg}$
 $\text{Eps_top} = 0.003500$
 $\text{Eps_bot} = -0.030196$
 $\text{Eps_stop} = 0.000692$
 $\text{Eps_sbot} = -0.027388$
- Results for reinforcement bars:
 $N_s = -420.79 \text{ e3}$
 $M_{xs} = 0.00 \text{ e3}$
 $M_{ys} = -216.50 \text{ e3}$
- Results for concrete section:
 $A_c = 0.0187$
 $(x_{cg}, y_{cg}) = (-0.27591769, 0.00000000)$
 $N_c = 420.64 \text{ e3}$
 $M_{xc} = 0.00 \text{ e3}$
 $M_{yc} = -116.06 \text{ e3}$

12.5.4 Results (manual calculations):

Verification of safety factor:

$$\begin{aligned} \alpha &= M_{yf} / M_y = 33.256 \\ N_f &= -0.16 \cong \alpha \cdot N = 0 \text{ (OK)} \\ M_{xf} &= \alpha \cdot M_x = 0 \text{ (OK)} \end{aligned}$$

We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.

Example Case 6b in this document presents calculations of internal forces for the section and stress-strain relation (both the same as in this example after rotation -90deg), and with the same material parameters and extreme strains. Appropriate internal forces and strains in the section calculated there are the same as a corresponding results from Component for this example.

Small differences between results from Component and from manual calculations are below 0.05% and are due to precision of iterations.

13 Case 12: Calculation of capacity state in symmetric section for bending moment M_x with axial force

13.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, loads applied to the section (N , M_x , 0) and give state of strain and stress at the failure.

13.2 Summary:

This sample gives as results the internal forces for:

- A symmetric concrete section with reinforcement bars
- A linear model of concrete stress-strain relation (Case 12a)
- A bilinear model of concrete stress-strain relation (Case12b)
- A power-rectangular model of concrete stress-strain relation (Case12c)
- A horizontal branch model of steel
- A bending in x direction (M_x)
- An axial force (N).

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel
- The loads applied to the section

Based on input data, Component gives as output results:

- Results for reinforcement bars:
 - Axial force (N_s)
 - Moment with respect to x_{cc} axis (M_{xs})
 - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
 - Area of contour under compressive stress (A_c)
 - Center of gravity of stresses in the concrete (x_{cg} , y_{cg})
 - Axial force (N_c)
 - Moment with respect to x_{cc} axis (M_{xc})
 - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
 - Axial force at the failure related to acting forces ($N_f = \alpha \cdot N$)
 - Moment at the failure related to acting forces, with respect to x_{cc} axis ($M_{xf} = \alpha \cdot M_x$)
 - Moment at the failure related to acting forces, with respect to y_{cc} axis ($M_{yf} = \alpha \cdot M_y$)

Where alpha is a coefficient called "safety factor".
- Neutral axis position:
 - Distance between the axis and a gravity center of concrete gross section (dist)
 - Angle between horizontal axis and normal to neutral axis directed to the most tensioned fiber (Angle)
- Strains
 - Extreme strains in concrete (Eps_top, Eps_bot)
 - Extreme strains in steel (Eps_stop, Eps_sbot)

For these results, x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

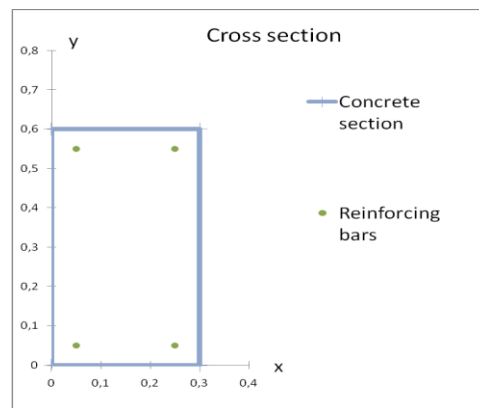
13.3 Case 12a

13.3.1 Data:

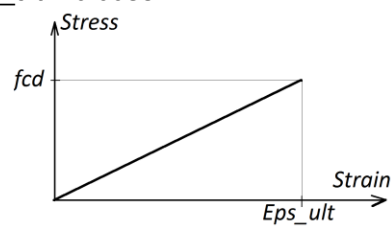
- Geometry:
 - Concrete section:

$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.6$
$x = 0.3$	$y = 0.6$
$x = 0.3$	$y = 0.0$
 - Reinforcing bars:

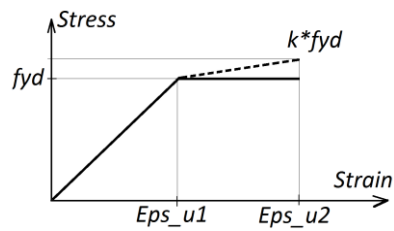
$x = 0.05$	$y = 0.05$	$\phi = 0.012$
$x = 0.05$	$y = 0.55$	$\phi = 0.012$
$x = 0.25$	$y = 0.55$	$\phi = 0.012$
$x = 0.25$	$y = 0.05$	$\phi = 0.012$



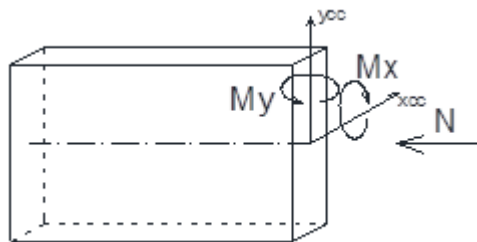
- Concrete parameters:
 - Design strength $f_{cd} = 20e6$
 - Modulus of elasticity $E_c = 30e9$
 - Strain-stress model: linear
 - Strain ultimate limit $Eps_ult = 0.0035$



- Steel parameters:
 - Design strength $f_{yd} = 500e6$
 - Hardening factor $k = 1.0$
 - Modulus of elasticity $E_s = 200e9$
 - Strain ultimate limit $Eps_u2 = 0.075$



- Forces:
 - $N = 43979.98 \text{ e3}$
 - $M_x = -3465.40 \text{ e3}$
 - $M_y = 0.00 \text{ e3}$



13.3.2 Search for:

- Results for reduced section:
 - Forces: N_f , M_{xf} , M_{yf}
 - Neutral axis position: dist, Angle
 - Extreme strains in concrete (Eps_top, Eps_bot)
 - Extreme strains in steel (Eps_stop, Eps_sbot)
- Results for reinforcement bars:
 - Forces: N_s , M_{xs} , M_{ys}
- Results for concrete section:
 - Area of contour under compressive stress: A_c
 - Center of gravity of stresses in the concrete: (x_{cg}, y_{cg})
 - Forces: N_c , M_{xc} , M_{yc}

13.3.3 Results from Component

- Results for reduced section:
 - $N_f = 2199.00 \text{ e3}$
 - $M_{xf} = -173.27 \text{ e3}$
 - $M_{yf} = 0.00 \text{ e3}$
 - dist = 0.40000
 - Angle = 4.712389= 270.00000deg
 - Eps_top = 0.003500
 - Eps_bot = 0.000500
 - Eps_stop = 0.003250
 - Eps_sbot = 0.000750

- Results for reinforcement bars:
 $N_s = 147.03 \text{ e3}$
 $M_{xs} = -19.79 \text{ e3}$
 $M_{ys} = 0.00 \text{ e3}$
- Results for concrete section:
 $A_c = 0.18$
 $(x_{cg}, y_{cg}) = (0.00000000, -0.07479510)$
 $N_c = 2051.97 \text{ e3}$
 $M_{xc} = 153.48 \text{ e3}$
 $M_{yc} = 0.00 \text{ e3}$

13.3.4 Results (manual calculations):

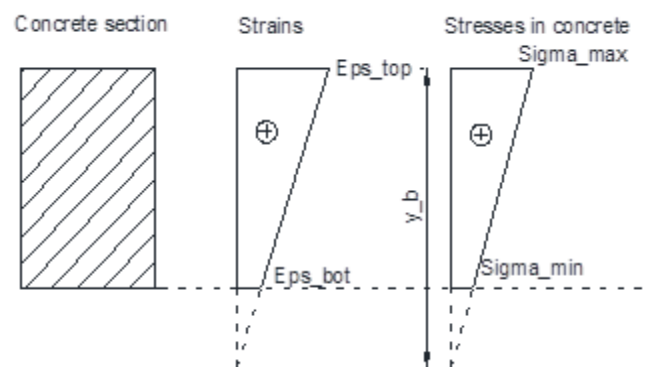
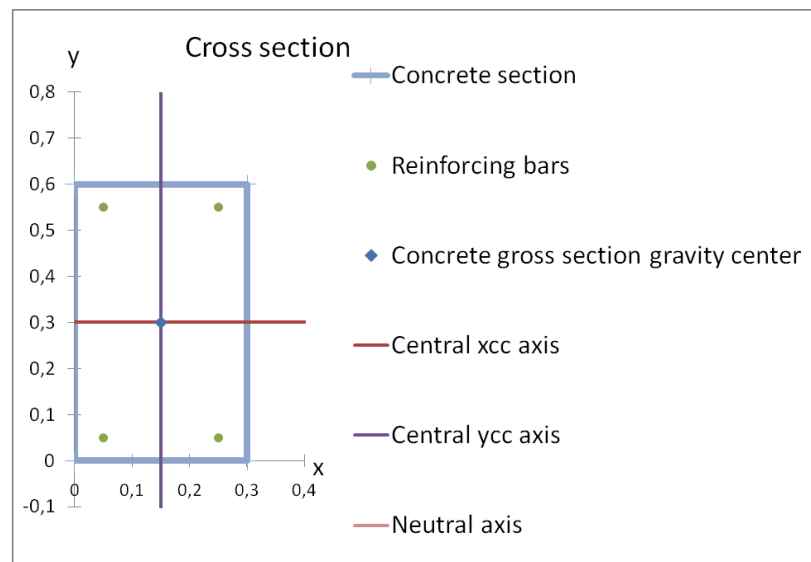
Verification of safety factor:

$$\alpha = M_{xf} / M_x = 0.05000$$

$$N_f = \alpha \cdot N = 2199.00 \text{ (OK)}$$

$$M_{yf} = \alpha \cdot M_y = 0 \text{ (OK)}$$

We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.



- Results for reinforcement bars:

Distance between maximum compression corner of the section and neutral axis:

height of a section: $h = 0.6$

$y_b = -Eps_top \cdot h / (Eps_bot - Eps_top) = 0.7$

$Eps_u1 = f_{yd} / E_s$

Concrete gross section gravity center:

$x_{Cc} = 0.15$

$y_{Cc} = 0.30$

Distance between reinforcing bar and neutral axis:

$d = y - h + y_b$

Stress in bar i:

If $|eps_i| \leq f_{yd} / E_s$ then

$\sigma_i = eps_i / Eps_u1 \cdot f_{yd}$

else

$\sigma_i = f_{yd} + (|eps_i| - Eps_u1) / (Eps_u2 - Eps_u1) \cdot (k - 1) \cdot f_{yd}$

i	x	y	ϕ	Distance d from neutral axis	strain $eps = d /$ $y_b \cdot$ Eps_top	stresses $\sigma_i /$ $10e6$	force in bar $N = \sigma_i \cdot \pi \cdot \phi^2 / 4$	Positions in central axes coordinate system		$M_{xs} = -$ $N \cdot y_{Cc}$	$M_{ys} =$ $N \cdot x_{Cc}$
								x_{Cc}	y_{Cc}		
1	0,05	0,05	0,012	0,150	0,00075	150	16965	-0,1	-0,25	4241	-1696
2	0,05	0,55	0,012	0,650	0,00325	500	56549	-0,1	0,25	-14137	-5655
3	0,25	0,55	0,012	0,650	0,00325	500	56549	0,1	0,25	-14137	5655
4	0,25	0,05	0,012	0,150	0,00075	150	16965	0,1	-0,25	4241	1696
						Sum of forces in bars:	147027			-19792	0

$N_s = 147.03e3$

$M_{xs} = -19.79e3$

$M_{ys} = 0.00e3$

$Eps_stop = \max(eps) = 0.00325$

$Eps_sbot = \min(eps) = 0.00075$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

i	x	y	ϕ	strain eps	Positions in central axes coordinate system		Area of concrete with compression stress cut from the section in place of bars	Force N_{sc} in concrete in place of bars $N_{sc} = f_{cd} \cdot eps$ / $Eps_ult \cdot Area$	$M_{xsc} = -$ $N_{sc} \cdot y_{Cc}$	$M_{ysc} =$ $N_{sc} \cdot x_{Cc}$
					x_{Cc}	y_{Cc}				
1	0,05	0,05	0,012	0,00075	-0,1	-0,25	0,000113097	485	121	-48
2	0,05	0,55	0,012	0,00325	-0,1	0,25	0,000113097	485	121	48

3	0,25	0,55	0,012	0,00325	0,1	0,25	0,000113097	2100	-525	210
4	0,25	0,05	0,012	0,00075	0,1	-0,25	0,000113097	2100	-525	-210
Sum of forces:								5170	-808	0

Reduction forces in concrete:

$$N_{sc} = 5.17e3$$

$$M_{xsc} = -0.81e3$$

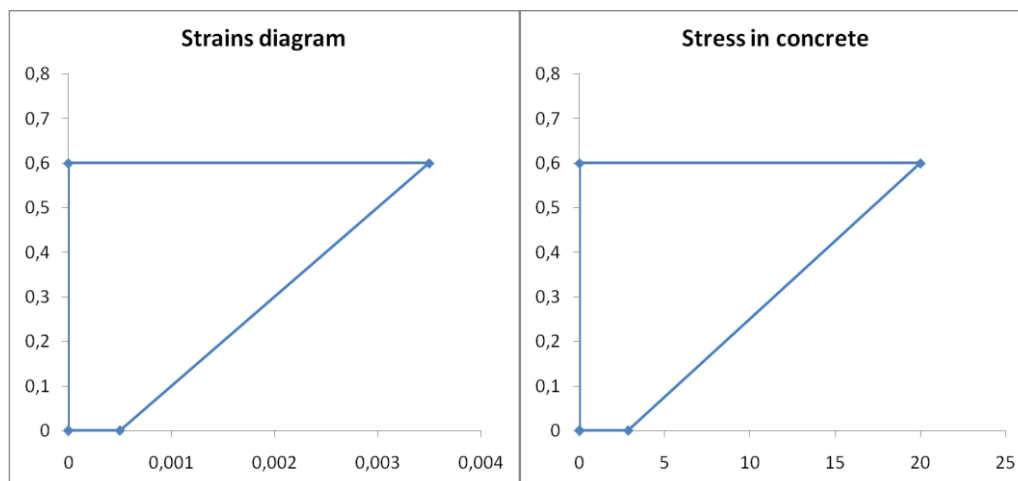
$$M_{ysc} = 0.00e3$$

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

$$x_{scg} = \sum(x_{cc_i} \cdot Area_i) / \sum(Area_i) = 0.00000$$

$$y_{scg} = \sum(y_{cc_i} \cdot Area_i) / \sum(Area_i) = 0.15625$$

- Results for concrete section:



Height of compressed part of the section:

$$0.6000$$

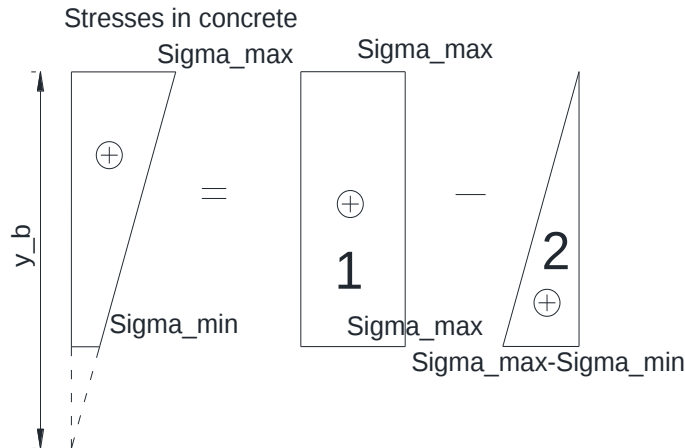
Area of contour under compressive stress:

$$A_c = 0.3 \cdot 0.6000 = 0.1800$$

Axial force N_c in compressed part of concrete section is calculated by subtraction of volume of a triangular solid of compressive stresses (2) from rectangular solid of compressive stresses (1) (see picture below and definition of N_c in Introduction).

$$N_{c'} = N_{c1} - N_{c2}$$

$$N_c = N_{c'} - N_{sc}$$



Similar approach is used for a center of gravity of stresses calculations, but additionally sums of stresses in solids are used as wages.

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc})$$

Moments M_{xc} and M_{yc} caused by stresses acting on entire compressed gross area are calculated following below equations:

$$M_{xc} = - N_c \cdot y_{cg}$$

$$M_{yc} = N_c \cdot x_{cg}$$

$$\text{Sigma_max} = f_{cd} \cdot \text{Eps_top} / \text{Eps_c1} = 20.000\text{e6}$$

$$\text{Sigma_min} = f_{cd} \cdot \text{Eps_bot} / \text{Eps_c1} = 2.857\text{e6}$$

Axial force in compressed part of concrete gross section

$$N_{c1} = A_c \cdot \text{Sigma_max} = 3.600\text{e6}$$

$$N_{c2} = A_c \cdot (\text{Sigma_max} - \text{Sigma_min}) / 2 = 1.543\text{e6}$$

$$N_c' = N_{c1} - N_{c2} = 2.057\text{e6}$$

Centers of gravity of stress solids 1 and 2 in central axes coordinate system:

$$x_{c1} = 0$$

$$y_{c1} = 0$$

$$x_{c2} = 0$$

$$y_{c2} = - y_{cc} + 0.6/3 = -0.1$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = (x_{c1} \cdot N_{c1} - x_{c2} \cdot N_{c2} - x_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.000000$$

$$y_{cg} = (y_{c1} \cdot N_{c1} - y_{c2} \cdot N_{c2} - y_{scg} \cdot N_{sc}) / (N_{c1} - N_{c2} - N_{sc}) = 0.074795$$

Internal forces in concrete:

$$N_c = N_c' - N_{sc} = 2051.97\text{e3}$$

$$M_{xc} = - N_c \cdot y_{cg} = -153.48\text{e3}$$

$$M_{yc} = N_c \cdot x_{cg} = 0.00\text{e3}$$

- Results for reduced section:

$$N_f = N_c + N_s = 2199.00\text{e3}$$

$$M_{xf} = M_{xc} + M_{xs} = -173.27\text{e3}$$

$$M_{yf} = M_{yc} + M_{ys} = 0.00\text{e3}$$

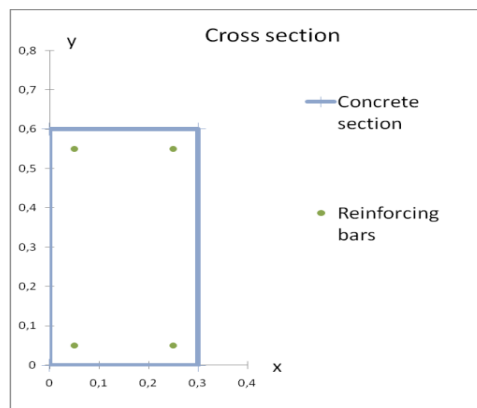
13.4 Case 12b

13.4.1 Data:

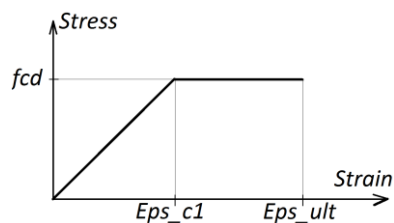
- Geometry:
 - Concrete section:

$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.6$
$x = 0.3$	$y = 0.6$
$x = 0.3$	$y = 0.0$
 - Reinforcing bars:

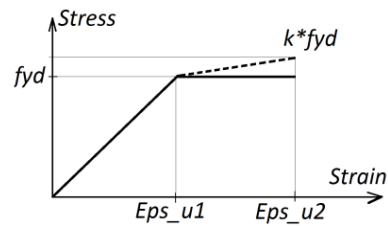
$x = 0.05$	$y = 0.05$	$\phi = 0.016$
$x = 0.05$	$y = 0.55$	$\phi = 0.016$
$x = 0.25$	$y = 0.55$	$\phi = 0.016$
$x = 0.25$	$y = 0.05$	$\phi = 0.016$



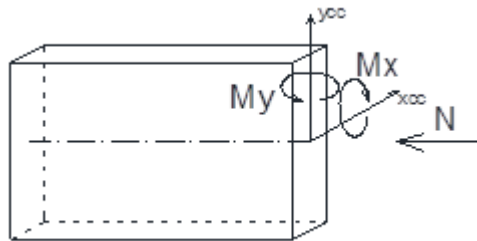
- Concrete parameters:
 - Design strength $f_{cd} = 30e6$
 - Modulus of elasticity $E_c = 32e9$
 - Strain-stress model: bilinear
 - Strain ultimate limit $Eps_ult = 0.0035$
 - Strain relation change over $Eps_c1 = 0.0020$



- Steel parameters:
 - Design strength $f_{yd} = 400e6$
 - Hardening factor $k = 1.0$
 - Modulus of elasticity $E_s = 205e9$
 - Strain ultimate limit $Eps_u2 = 0.1$



- Forces:
 - $N = 114.8397 \text{ e3}$
 - $M_x = -5.634275 \text{ e3}$
 - $M_y = 0.00 \text{ e3}$



13.4.2 Search for:

- Results for reduced section:
 - Forces: N_f , M_{xf} , M_{yf}
 - Neutral axis position: dist, Angle
 - Extreme strains in concrete (Eps_top, Eps_bot)
 - Extreme strains in steel (Eps_stop, Eps_sbot)
- Results for reinforcement bars:
 - Forces: N_s , M_{xs} , M_{ys}
- Results for concrete section:
 - Area of contour under compressive stress: A_c
 - Center of gravity of stresses in the concrete: (x_{cg}, y_{cg})
 - Forces: N_c , M_{xc} , M_{yc}

13.4.3 Results from Component

- Results for reduced section:
 - $N_f = 4593.59 \text{ e3}$
 - $M_{xf} = -225.37 \text{ e3}$
 - $M_{yf} = 0.00 \text{ e3}$
 - dist = 0.40000
 - Angle = 4.71239= 270 deg
 - Eps_top = 0.003500
 - Eps_bot = 0.000500
 - Eps_stop = 0.003250
 - Eps_sbot = 0.000750
- Results for reinforcement bars:

$$N_s = 222.68 \text{ e3}$$

$$M_{xs} = -24.76 \text{ e3}$$

$$M_{ys} = 0.00 \text{ e3}$$

- Results for concrete section:
 $A_c = 0.1800$
 $(x_{cg}, y_{cg}) = (0.00000000, 0.04589755)$
 $N_c = 4370.92 \text{ e3}$
 $M_{xc} = -200.61 \text{ e3}$
 $M_{yc} = 0.00 \text{ e3}$

13.4.4 Results (manual calculations):

Verification of safety factor:

$$\alpha = M_{xt} / M_x = 40$$

$$N_f = \alpha \cdot N = 4593.59 \text{ (OK)}$$

$$M_{yt} = \alpha \cdot M_y = 0 \text{ (OK)}$$

We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.

The same neutral axis position, extreme strains and material properties were used as data for Case 4a in this document. Internal forces and strains in the section calculated there are the same as results from Component for this example.

Small differences between results from Component and from manual calculations are below 0.01% and are due to precision of iterations.

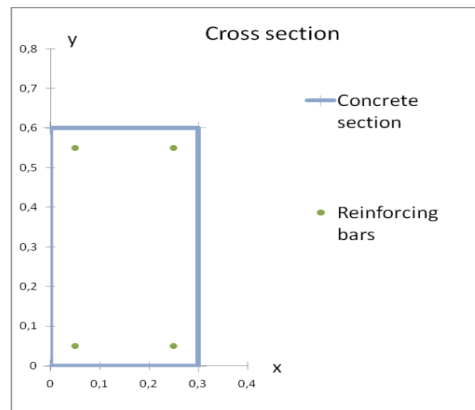
13.5 Case 12c

13.5.1 Data:

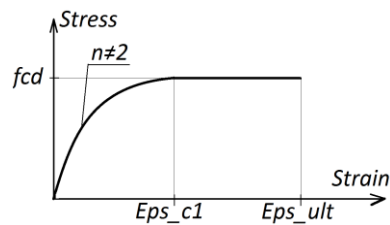
- Geometry:
 - Concrete section:

$x = 0.0$	$y = 0.0$	
$x = 0.0$	$y = 0.6$	
$x = 0.3$	$y = 0.6$	
$x = 0.3$	$y = 0.0$	
 - Reinforcing bars:

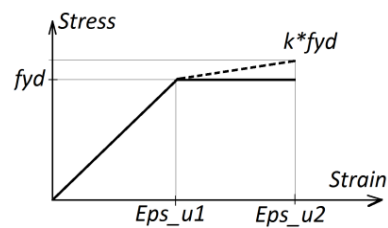
$x = 0.05$	$y = 0.05$	$\phi = 0.016$
$x = 0.05$	$y = 0.55$	$\phi = 0.016$
$x = 0.25$	$y = 0.55$	$\phi = 0.016$
$x = 0.25$	$y = 0.05$	$\phi = 0.016$



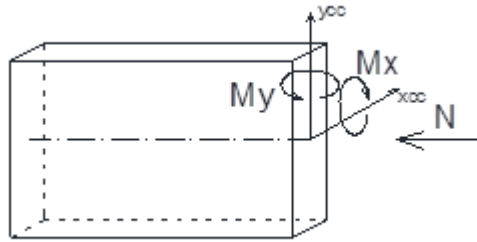
- Concrete parameters:
 - Design strength $f_{cd} = 30e6$
 - Modulus of elasticity $E_c = 32e9$
 - Strain-stress model: power-rectangular
 - Power: $n = 1.4$
 - Strain ultimate limit $Eps_ult = 0.0035$
 - Strain relation change over $Eps_c1 = 0.0020$



- Steel parameters:
 - Design strength $f_{yd} = 400e6$
 - Hardening factor $k = 1.0$
 - Modulus of elasticity $E_s = 200e9$
 - Strain ultimate limit $Eps_u2 = 0.1$



- Forces:
 - $N = 4.41023 \text{ e}3$
 - $M_x = -0.1662045455 \text{ e}3$
 - $M_y = 0.00 \text{ e}3$



13.5.2 Search for:

- Results for reduced section:
 - Forces: N_f , M_{xf} , M_{yf}
 - Neutral axis position: dist, Angle
 - Extreme strains in concrete (Eps_top, Eps_bot)
 - Extreme strains in steel (Eps_stop, Eps_sbot)
- Results for reinforcement bars:
 - Forces: N_s , M_{xs} , M_{ys}
- Results for concrete section:
 - Area of contour under compressive stress: A_c
 - Center of gravity of stresses in the concrete: (x_{cg}, y_{cg})
 - Forces: N_c , M_{xc} , M_{yc}

13.5.3 Results from Component

- Results for reduced section:
 - $N_f = 4851.25e3$
 - $M_{xf} = -182.83e3$
 - $M_{yf} = 0.00e3$
 - dist = 0.40000
 - Angle = 4.71239= 270deg
 - Eps_top = 0.003500
 - Eps_bot = 0.000500
 - Eps_stop = 0.003250
 - Eps_sbot = 0.000750
- Results for reinforcement bars:
 - $N_s = 221.17e3$
 - $M_{xs} = -25.13e3$
 - $M_{ys} = 0.00e3$
- Results for concrete section:
 - $A_c = 0.1800$
 - $(x_{cg}, y_{cg}) = (0.00000000, 0.03405830)$
 - $N_c = 4630.08e3$
 - $M_{xc} = -157.69e3$
 - $M_{yc} = 0.00e3$

13.5.4 Results (manual calculations):

Verification of safety factor:

$$\alpha = M_{xf} / M_x = 1100$$

$$N_f = \alpha \cdot N = 4851.25 \text{ (OK)}$$

$$M_{yf} = \alpha \cdot M_y = 0 \text{ (OK)}$$

We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.

The same neutral axis position, extreme strains and material properties were used as data for Case 6a in this document. Internal forces and strains in the section calculated there are the same as results from Component for this example.

Small differences between results from Component and from manual calculations are below 0.01% and are due to precision of iterations.

14 Case 13: Calculation of capacity state in symmetric section for bending moment M_y with axial force

14.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, loads applied to the section (N , 0 , M_y) and give state of strain and stress at the failure.

14.2 Summary:

This sample gives as results the internal forces for:

- A symmetric concrete section with reinforcement bars
- A rectangular model of concrete stress-strain relation (Case 13a)
- A parabolic-rectangular model of concrete stress-strain relation (Case13b)
- A horizontal branch model of steel
- A bending in y direction (M_y)
- An axial force (N).

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel
- The loads applied to the section

Based on input data, Component gives as output results:

- Results for reinforcement bars:
 - Axial force (N_s)
 - Moment with respect to x_{cc} axis (M_{xs})
 - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
 - Area of contour under compressive stress (A_c)
 - Center of gravity of stresses in the concrete (x_{cg} , y_{cg})
 - Axial force (N_c)
 - Moment with respect to x_{cc} axis (M_{xc})
 - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
 - Axial force at the failure related to acting forces ($N_f = \alpha \cdot N$)
 - Moment at the failure related to acting forces, with respect to x_{cc} axis ($M_{xf} = \alpha \cdot M_x$)
 - Moment at the failure related to acting forces, with respect to y_{cc} axis ($M_{yf} = \alpha \cdot M_y$)

Where α is a coefficient called "safety factor".
- Neutral axis position:
 - Distance between the axis and a gravity center of concrete gross section (dist)
 - Angle between horizontal axis and normal to neutral axis directed to the most tensioned fiber (Angle)
- Strains
 - Extreme strains in concrete (Eps_top, Eps_bot)
 - Extreme strains in steel (Eps_stop, Eps_sbot)

Where x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

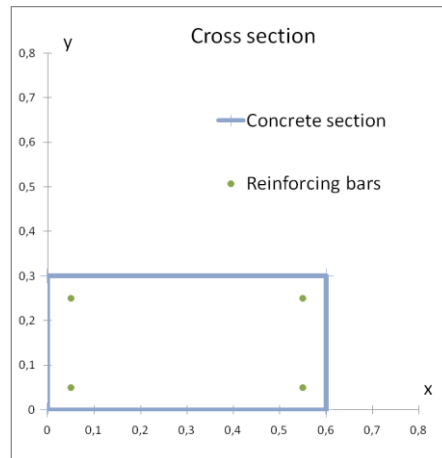
14.3 Case 13a

14.3.1 Data:

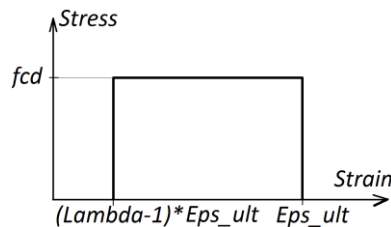
- Geometry:
 - Concrete section:

$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.3$
$x = 0.6$	$y = 0.3$
$x = 0.6$	$y = 0.0$
 - Reinforcing bars:

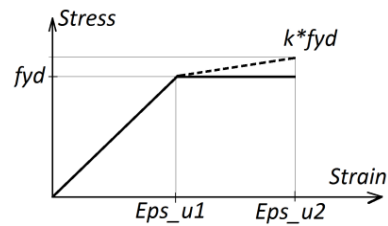
$x = 0.05$	$y = 0.05$	$\phi = 0.012$
$x = 0.05$	$y = 0.25$	$\phi = 0.012$
$x = 0.55$	$y = 0.25$	$\phi = 0.012$
$x = 0.55$	$y = 0.05$	$\phi = 0.012$



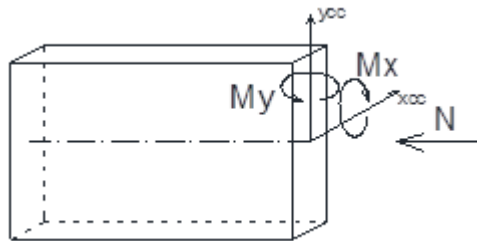
- Concrete parameters:
 - Design strength $f_{cd} = 20e6$
 - Effective height reduction factor $\Lambda = 0.8$
 - Modulus of elasticity $E_c = 30e9$
 - Strain-stress model: rectangular
 - Strain ultimate limit $Eps_ult = 0.0035$



- Steel parameters:
 - Design strength $f_{yd} = 500e6$
 - Hardening factor $k = 1.0$
 - Modulus of elasticity $E_s = 200e9$
 - Strain ultimate limit $Eps_u2 = 0.075$



- Forces:
 - $N = 3331.408571 \text{ e3}$
 - $M_x = 0.00 \text{ e3}$
 - $M_y = -82.84952381 \text{ e3}$



14.3.2 Search for:

- Results for reduced section:
 - Forces: N_f , M_{xf} , M_{yf}
 - Neutral axis position: dist, Angle
 - Extreme strains in concrete (Eps_top, Eps_bot)
 - Extreme strains in steel (Eps_stop, Eps_sbot)
- Results for reinforcement bars:
 - Forces: N_s , M_{xs} , M_{ys}
- Results for concrete section:
 - Area of contour under compressive stress: A_c
 - Center of gravity of stresses in the concrete: (x_{cg}, y_{cg})
 - Forces: N_c , M_{xc} , M_{yc}

14.3.3 Results from Component

- Results for reduced section:
 - $N_f = 3497.98 \text{ e3}$
 - $M_{xf} = 0.00 \text{ e3}$
 - $M_{yf} = -86.99 \text{ e3}$
 - dist = 0.400000
 - Angle = 0.00000 = 0.0000deg
 - Eps_top = 0.003500
 - Eps_bot = 0.000500
 - Eps_stop = 0.003250
 - Eps_sbot = 0.000750
- Results for reinforcement bars:
 - $N_s = 147.03 \text{ e3}$
 - $M_{xs} = 0.00 \text{ e3}$
 - $M_{ys} = -19.79 \text{ e3}$

- Results for concrete section:
 $A_c = 0.1680$
 $(x_{cg}, y_{cg}) = (-0.02005403, 0.00000000)$
 $N_c = 3350.95 \text{ e3}$
 $M_{xc} = 0.00 \text{ e3}$
 $M_{yc} = -67.20 \text{ e3}$

14.3.4 Results (manual calculations):

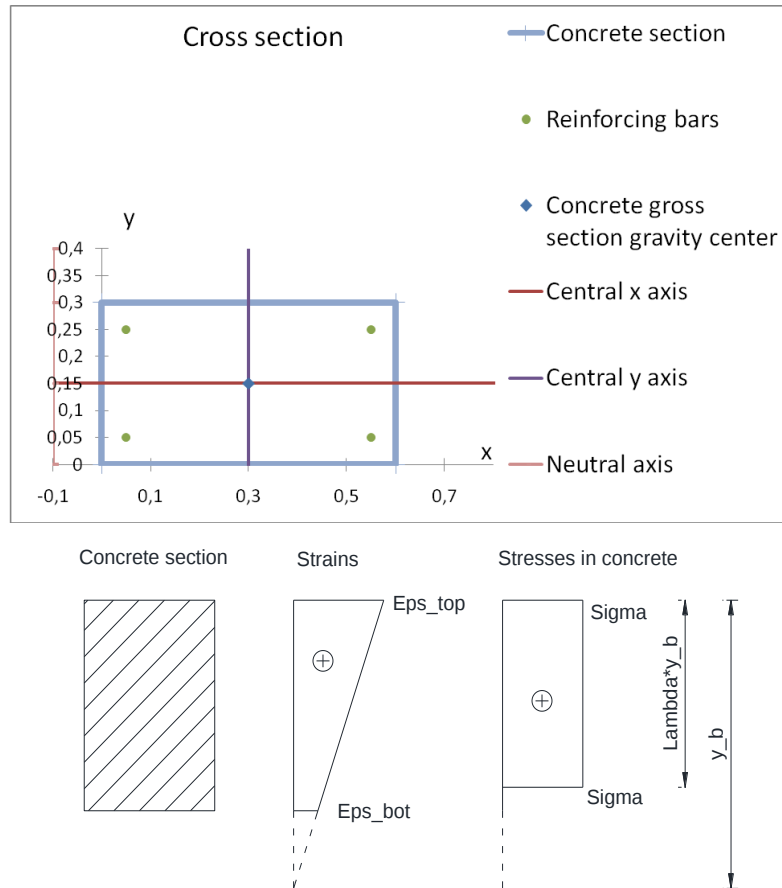
Verification of safety factor:

$$\alpha = M_{yf} / M_y = 1.04998$$

$$N_f = 3497.98 \cong \alpha \cdot N = 3497.90 \text{ (OK)}$$

$$M_{xf} = \alpha \cdot M_y = 0 \text{ (OK)}$$

We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.



- Results for reinforcement bars:

Distance between maximum compression corner of the section and neutral axis:

width of a section: $b = 0.6$

$$y_b = -Eps_{top} \cdot b / (Eps_{bot} - Eps_{top}) = 0.7$$

$$Eps_{u1} = f_{yd} / E_s$$

Concrete gross section gravity center:

$$x_{Cc} = 0.30$$

$$y_{Cc} = 0.15$$

Distance between reinforcing bar and neutral axis:

$$d = y - b + y_b$$

Stress in bar i:

If $|\epsilon_{ps_i}| \leq f_{yd} / E_s$ then

$$\sigma_{ps_i} = \epsilon_{ps_i} / \epsilon_{ps_u1} \cdot f_{yd}$$

else

$$\sigma_{ps_i} = f_{yd} + (|\epsilon_{ps_i}| - \epsilon_{ps_u1}) / (\epsilon_{ps_u2} - \epsilon_{ps_u1}) \cdot (k - 1) \cdot f_{yd}$$

i	x	y	ϕ	Distance d from neutral axis	strain $\epsilon_{ps} = d /$ $y_b \cdot$ ϵ_{ps_top}	stresses $\sigma_{ps} /$ 10e6	force in bar $N = \sigma_{ps} \cdot$ $\pi \cdot \phi^2 / 4$	Positions in central axes coordinate system x_{Cc} y_{Cc}		$M_{xs} = -$ $N \cdot y_{Cc}$	$M_{ys} = N$ $\cdot x_{Cc}$
1	0,05	0,05	0,012	0,150	0,00325	500	56549	-0,25	-0,10	5655	-14137
2	0,55	0,05	0,012	0,650	0,00075	150	16965	0,25	-0,10	1696	4241
3	0,55	0,25	0,012	0,650	0,00075	150	16965	0,25	0,10	-1696	4241
4	0,05	0,25	0,012	0,150	0,00325	500	56549	-0,25	0,10	-5655	-14137
						Sum of forces in bars:	147027			- 19792	0

$$N_s = 147.03e3$$

$$M_{xs} = -19.79e3$$

$$M_{ys} = 0.00e3$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

i	x	y	ϕ	Positions in central axes coordinate system x_{Cc} y_{Cc}		Area of concrete with compression stress cut from the section in place of bars	Force N_{sc} in concrete in place of bars $N_{sc} = f_{cd} \cdot$ $\epsilon_{ps_top} / \epsilon_{ps_ult} \cdot$ Area	$M_{xsc} = -$ $N_{sc} \cdot y_{Cc}$	$M_{ysc} =$ $N_{sc} \cdot x_{Cc}$
1	0,05	0,05	0,012	-0,25	-0,10	0,000113097	2262	226	-565
2	0,55	0,05	0,012	0,25	-0,10	0,000113097	2262	226	565
3	0,55	0,25	0,012	0,25	0,10	0,000113097	2262	-226	565
4	0,05	0,25	0,012	-0,25	0,10	0,000113097	2262	-226	-565
						Sum of forces:	9048	0	0

Reduction forces in concrete:

$$N_{sc} = 9.05e3$$

$$M_{xsc} = 0.00e3$$

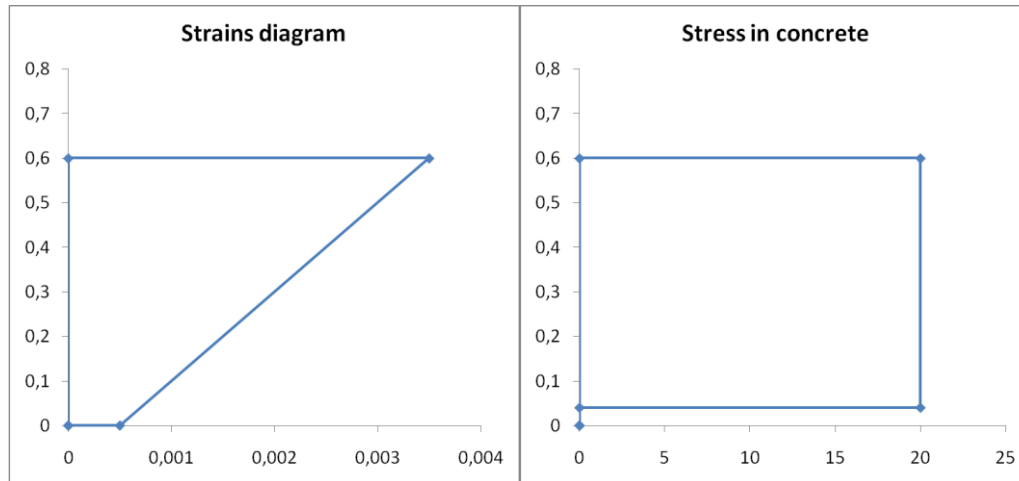
$$M_{ysc} = 0.00e3$$

Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

$$x_{scg} = \sum(x_{cc_i} \cdot Area_i) / \sum(Area_i) = 0.00$$

$$y_{scg} = \sum(y_{cc_i} \cdot Area_i) / \sum(Area_i) = 0.00$$

- Results for concrete section:



Height of compressed part of the section:

$$\text{Lambda} \cdot y_b = 0.56$$

Area of contour under compressive stress:

$$A_c = 0.3 \cdot 0.56 = \underline{0.168}$$

Stress in concrete:

$$\text{Sigma} = \text{Eps_top} / \text{Eps_ult} \cdot f_{cd} = 20\text{e}6$$

Center of gravity of stresses in the concrete in central axes coordinate system:

$$x_{cg} = ((0.56 / 2 - x_{cc}) \cdot A_c \cdot \text{Sigma} - x_{scg} \cdot N_{sc}) / (A_c \cdot \text{Sigma} + N_{sc}) = \underline{-0.020054}$$

$$y_{cg} = ((0.3 / 2 - y_{cc}) \cdot A_c \cdot \text{Sigma} - y_{scg} \cdot N_{sc}) / (A_c \cdot \text{Sigma} + N_{sc}) = \underline{0.000000}$$

Internal forces in concrete:

$$N_c = A_c \cdot \text{Sigma} - N_{sc} = \underline{3350.95\text{e}3}$$

$$M_{xc} = -N_c \cdot y_{cg} = \underline{0.00\text{e}3}$$

$$M_{yc} = N_c \cdot x_{cg} = \underline{-67.20\text{e}3}$$

- Results for reduced section:

$$N_f = N_c + N_s = \underline{3497.98\text{e}3}$$

$$M_{xf} = M_{xc} + M_{xs} = \underline{0.00\text{e}3}$$

$$M_{yf} = M_{yc} + M_{ys} = \underline{-86.99\text{e}3}$$

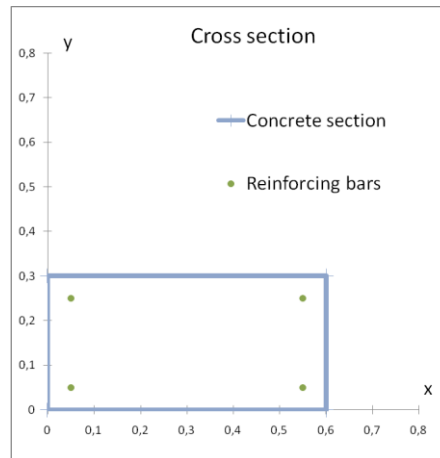
14.4 Case 13b

14.4.1 Data:

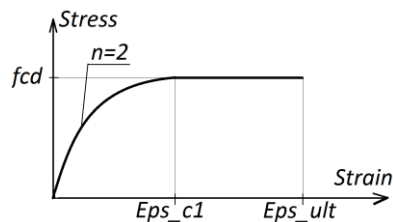
- Geometry:
 - Concrete section:

$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.3$
$x = 0.6$	$y = 0.3$
$x = 0.6$	$y = 0.0$
 - Reinforcing bars:

$x = 0.05$	$y = 0.05$	$\phi = 0.016$
$x = 0.05$	$y = 0.25$	$\phi = 0.016$
$x = 0.55$	$y = 0.25$	$\phi = 0.016$
$x = 0.55$	$y = 0.05$	$\phi = 0.016$

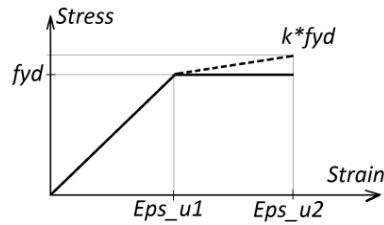


- Concrete parameters:
 - Design strength $f_{cd} = 30e6$
 - Modulus of elasticity $E_c = 32e9$
 - Strain-stress model: parabolic-rectangular
 - Strain ultimate limit $Eps_ult = 0.0035$
 - Strain relation change over $Eps_c1 = 0.0020$

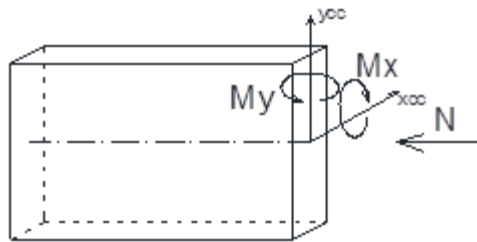


- Steel parameters:
 - Design strength $f_{yd} = 400e6$
 - Hardening factor $k = 1.0$
 - Modulus of elasticity $E_s = 200e9$

- Strain ultimate limit $Eps_u2 = 0.1$



- Forces:
 - $N = 5095.50e3$
 - $M_x = 0.00e3$
 - $M_y = -137.86e3$



14.4.2 Search for:

- Results for reduced section:
 - Forces: N_f , M_{xf} , M_{yf}
 - Neutral axis position: dist, Angle
 - Extreme strains in concrete (Eps_top , Eps_bot)
 - Extreme strains in steel (Eps_stop , Eps_sbot)
- Results for reinforcement bars:
 - Forces: N_s , M_{xs} , M_{ys}
- Results for concrete section:
 - Area of contour under compressive stress: A_c
 - Center of gravity of stresses in the concrete: (x_{cg} , y_{cg})
 - Forces: N_c , M_{xc} , M_{yc}

14.4.3 Results from Component

- Results for reduced section:
 - $N_f = 5095.50 \text{ e3}$
 - $M_{xf} = 0.00 \text{ e3}$
 - $M_{yf} = -137.86 \text{ e3}$
 - dist = 0.400000
 - Angle = 0.00000 = 0.0000deg
 - $Eps_top = 0.003500$
 - $Eps_bot = 0.000500$
 - $Eps_stop = 0.003250$
 - $Eps_sbot = 0.000750$
- Results for reinforcement bars:
 - $N_s = 221.17 \text{ e3}$
 - $M_{xs} = 0.00 \text{ e3}$
 - $M_{ys} = -25.13 \text{ e3}$

- Results for concrete section:
 $A_c = 0.1800$
 $(x_{cg}, y_{cg}) = (-0.02312690, 0.000000000)$
 $N_c = 4874.33 \text{ e3}$
 $M_{xc} = 0.00 \text{ e3}$
 $M_{yc} = -112.73 \text{ e3}$

14.4.4 Results (manual calculations):

Verification of safety factor:

$$\begin{aligned}\alpha &= M_{yf} / M_y = 1.000 \\ N_f &= \alpha \cdot N = 5095.50 \text{ (OK)} \\ M_{xf} &= \alpha \cdot M_x = 0 \text{ (OK)}\end{aligned}$$

We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.

Example Case 5a in this document presents calculations of internal forces for the section and stress-strain relation (both the same as in this example after rotation -90deg), and with the same material parameters and extreme strains. Appropriate internal forces and strains in the section calculated there are the same as a corresponding results from Component for this example.

Small differences between results from Component and from manual calculations are below 0.01% and are due to precision of iterations.

15 Case 14: Calculation of capacity state in symmetric section for bidirectional bending with axial force

15.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, loads applied to the section (N , M_x , M_y) and give state of strain and stress at the failure.

15.2 Summary:

This sample gives as results the internal forces for:

- A symmetric concrete section with reinforcement bars
- A rectangular model of concrete stress-strain relation
- A horizontal branch model of steel
- A bending in x direction (M_x)
- A bending in y direction (M_y)
- An axial force (N).

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel
- The loads applied to the section

Based on input data, Component gives as output results:

- Results for reinforcement bars:
 - Axial force (N_s)
 - Moment with respect to x_{cc} axis (M_{xs})
 - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
 - Area of contour under compressive stress (A_c)
 - Center of gravity of stresses in the concrete (x_{cg} , y_{cg})
 - Axial force (N_c)
 - Moment with respect to x_{cc} axis (M_{xc})
 - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
 - Axial force at the failure related to acting forces ($N_f = \alpha \cdot N$)
 - Moment at the failure related to acting forces, with respect to x_{cc} axis ($M_{xf} = \alpha \cdot M_x$)
 - Moment at the failure related to acting forces, with respect to y_{cc} axis ($M_{yf} = \alpha \cdot M_y$)

Where alpha is a coefficient called "safety factor".
- Neutral axis position:
 - Distance between the axis and a gravity center of concrete gross section (dist)
 - Angle between horizontal axis and normal to neutral axis directed to the most tensioned fiber (Angle)
- Strains
 - Extreme strains in concrete (Eps_top, Eps_bot)
 - Extreme strains in steel (Eps_stop, Eps_sbot)

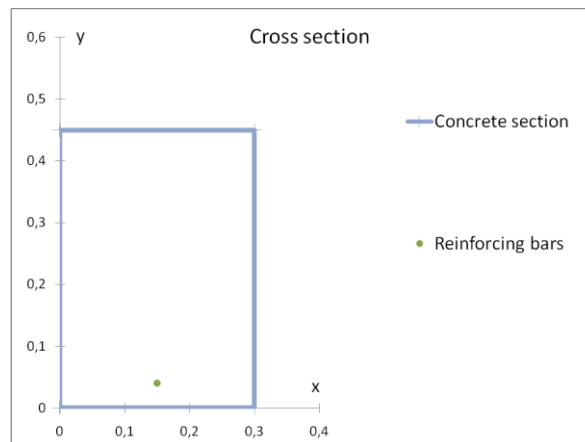
Where x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

15.3 Data:

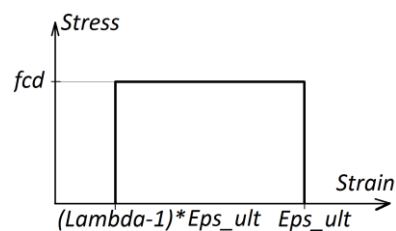
- Geometry:
 - Concrete section:

$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.45$
$x = 0.3$	$y = 0.45$
$x = 0.3$	$y = 0.0$
 - Reinforcing bars:

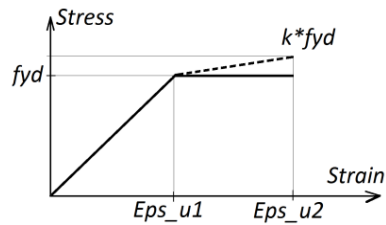
$x = 0.15$	$y = 0.04$	area= 0.00053	$\phi = 0.025977239$
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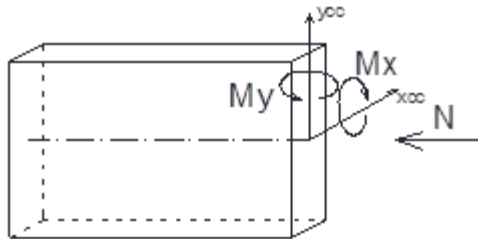
- Concrete parameters:
 - Design strength $f_{cd} = 26.7 \text{ e6} \cdot 0.8 = 21.3 \text{ e6}$
 - Effective height reduction factor $\text{Lambda} = 0.8$
 - Modulus of elasticity $E_c = 35\text{e9}$
 - Strain-stress model: rectangular
 - Strain ultimate limit $\text{Eps_ult} = 0.0035$



- Steel parameters:
 - Design strength $f_{yd} = 310\text{e6}$
 - Hardening factor $k = 1.0$
 - Modulus of elasticity $E_s = 200\text{e9}$
 - Strain ultimate limit $\text{Eps_u2} = 0.075$



- Forces:
 - $N = 200.00 \text{ e3}$
 - $M_x = -N \cdot 0.48 = -96.00 \text{ e3}$
 - $M_y = -N \cdot 0.12 = 24.00 \text{ e3}$



15.4 Search for:

- Results for reduced section:
 - Forces: N_f , M_{xf} , M_{yf}
 - Extreme strains in concrete (Eps_top, Eps_bot)
- Results for concrete section:
 - Area of contour under compressive stress: A_c
 - Force: N_c

15.5 Results from Component

- Results for reduced section:
 - $N_f = 199.77 \text{ e3}$
 - $M_{xf} = -95.95 \text{ e3}$
 - $M_{yf} = -24.00 \text{ e3}$
 - Angle = $5.204135 = 298.174942 \text{ deg}$
 - dist = -0.120248
 - Eps_top = 0.003500
 - Eps_bot = -0.009152
 - Eps_stop = -0.006659
 - Eps_sbot = -0.006659
- Results for reinforcement bars:
 - $N_s = -164.41 \text{ e3}$
 - $M_{xs} = -30.41 \text{ e3}$
 - $M_{ys} = 0.00 \text{ e3}$
- Results for concrete section:
 - $A_c = 0.0170$
 - $(x_{cg}, y_{cg}) = (-0.06589487, 0.17995110)$
 - $N_c = 364.018 \text{ e3}$

$$M_{xc} = -65.53 \text{ e3}$$

$$M_{yc} = -24.00 \text{ e3}$$

Results from Component are consistent with results from book *“Podstawy projektowania konstrukcji żelbetowych i sprężonych według Eurokodu 2”*, Sekcja Konstrukcji Betonowych KILiW PAN, Wrocław 2006, example 6.18 on page 352., ISBN 83-7125-136-X. Calculations in the book are performed with smaller precision than in Component which causes some differences in results (about 1%).

These results are also consistent with results from a program *spColumn v4.20*, STRUCTUREPOINT, LLC with precision over 99%.

16 Case 15: Calculation of capacity state in asymmetric section for bidirectional bending with axial force

16.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, loads applied to the section (N , M_x , M_y) and give state of strain and stress at the failure.

16.2 Summary:

This sample gives as results the internal forces for:

- A asymmetric concrete section with reinforcement bars
- A rectangular model of concrete stress-strain relation (Case 15a)
- A horizontal branch model of steel
- A bending in x direction (M_x)
- A bending in y direction (M_y)
- An axial force (N).

In the Case 15b can be seen a comparison of capacity forces for various models of concrete (rectangular, linear, bilinear, parabolic-rectangular and power-rectangular) obtained in Component and in manual calculations.

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel
- The loads applied to the section

Based on input data, Component gives as output results:

- Results for reinforcement bars:
 - Axial force (N_s)
 - Moment with respect to x_{cc} axis (M_{xs})
 - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
 - Area of contour under compressive stress (A_c)
 - Center of gravity of stresses in the concrete (x_{cg} , y_{cg})
 - Axial force (N_c)
 - Moment with respect to x_{cc} axis (M_{xc})
 - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
 - Axial force at the failure related to acting forces ($N_f = \alpha \cdot N$)
 - Moment at the failure related to acting forces, with respect to x_{cc} axis ($M_{xf} = \alpha \cdot M_x$)
 - Moment at the failure related to acting forces, with respect to y_{cc} axis ($M_{yf} = \alpha \cdot M_y$)

Where alpha is a coefficient called "safety factor".
- Neutral axis position:
 - Distance between the axis and a gravity center of concrete gross section (dist)
 - Angle between horizontal axis and normal to neutral axis directed to the most tension fiber.(Angle)
- Strains
 - Extreme strains in concrete (Eps_top, Eps_bot)

- Extreme strains in steel (Eps_stop, Eps_sbot)

Where x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

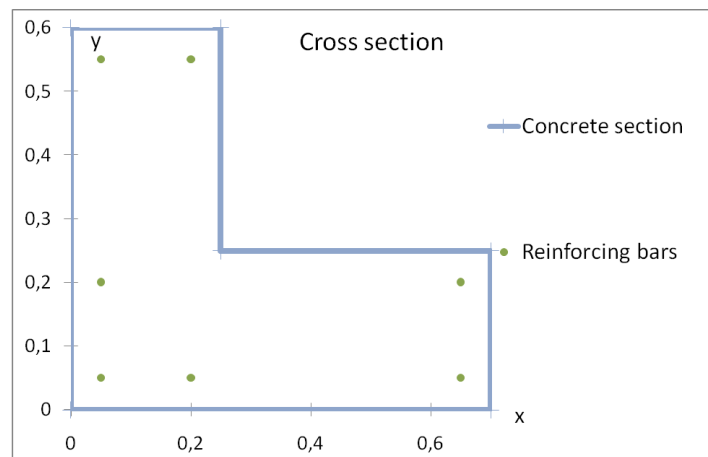
16.3 Case 15a

16.3.1 Data:

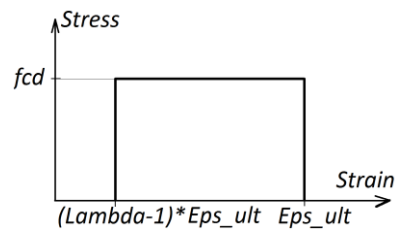
- Geometry:
 - Concrete section:

$x = 0.00$	$y = 0.00$
$x = 0.00$	$y = 0.60$
$x = 0.25$	$y = 0.60$
$x = 0.25$	$y = 0.25$
$x = 0.70$	$y = 0.25$
$x = 0.70$	$y = 0.00$
 - Reinforcing bars:

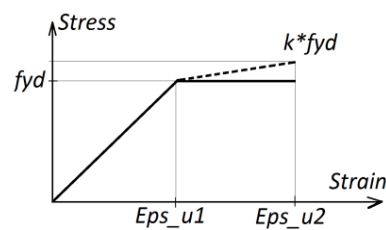
$x = 0.05$	$y = 0.05$	$\phi = 0.020$
$x = 0.05$	$y = 0.55$	$\phi = 0.020$
$x = 0.20$	$y = 0.55$	$\phi = 0.020$
$x = 0.20$	$y = 0.05$	$\phi = 0.020$
$x = 0.65$	$y = 0.05$	$\phi = 0.020$
$x = 0.65$	$y = 0.20$	$\phi = 0.020$
$x = 0.05$	$y = 0.20$	$\phi = 0.020$



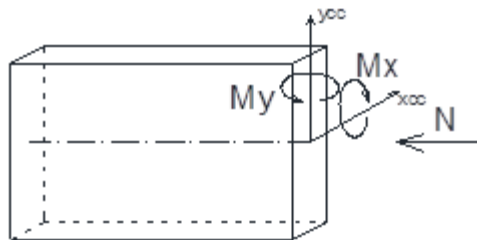
- Concrete parameters:
 - Design strength $f_{cd} = 20e6$
 - Effective height reduction factor $\Lambda = 0.8$
 - Modulus of elasticity $E_c = 35e9$
 - Strain-stress model: rectangular
 - Strain ultimate limit $Eps_{ult} = 0.0035$



- Steel parameters:
 - Design strength $f_{yd} = 310e6$
 - Hardening factor $k = 1.0$
 - Modulus of elasticity $E_s = 200e9$
 - Strain ultimate limit $Eps_u2 = 0.075$



- Forces:
 - $N = 72.4471\ e3$
 - $M_x = -28.9825\ e3$
 - $M_y = 2.5743\ e3$



16.3.2 Search for:

- Results for reduced section:
 - Forces: N_f , M_{xf} , M_{yf}
 - Neutral axis position: dist, Angle
 - Extreme strains in concrete (Eps_top , Eps_bot)
 - Extreme strains in steel (Eps_stop , Eps_sbot)
- Results for reinforcement bars:
 - Forces: N_s , M_{xs} , M_{ys}
- Results for concrete section:
 - Area of contour under compressive stress: A_c
 - Center of gravity of stresses in the concrete: (x_{cg} , y_{cg})
 - Forces: N_c , M_{xc} , M_{yc}

16.3.3 Results from Component

- Results for reduced section:
 $N_f = 725.91 \text{ e3}$
 $M_{xf} = -290.09 \text{ e3}$
 $M_{yf} = 25.68 \text{ e3}$
 $\text{Angle} = 4.188995 = 240.011739 \text{ deg}$
 $\text{dist} = 0.095114$
 $\text{Eps_top} = 0.003500$
 $\text{Eps_bot} = -0.006888$
 $\text{Eps_stop} = 0.002399$
 $\text{Eps_sbot} = -0.005788$
- Results for reinforcement bars:
 $N_s = -108.39 \text{ e3}$
 $M_{xs} = -101.21 \text{ e3}$
 $M_{ys} = 31.32 \text{ e3}$
- Results for concrete section:
 $A_c = 0.0427$
 $(x_{cg}, y_{cg}) = (-0.00676059, 0.22639273)$
 $N_c = 834.30 \text{ e3}$
 $M_{xc} = -188.88 \text{ e3}$
 $M_{yc} = -5.64 \text{ e3}$

16.3.4 Results (manual calculations):

Verification of safety factor:

$$M_{yf} / M_y = 9.98$$

$$M_{xf} / M_x = 10.01$$

$$N_f / N = 10.02$$

$$\alpha = M_{yf} / M_y \approx M_{xf} / M_x \approx N_f / N \text{ (OK)}$$

We will use neutral axis position and extreme strains obtained from Component and calculate forces in the section. They should be the same as from Component.

Position of neutral axis

Concrete gross section gravity center calculations:

i	Corners of concrete section		Area_c	Gravity center of contour corresponding to Area_c		Static moments	
	x	y		x_{01}	y_{01}	S_{cy}	S_{cx}
1	0	0	0,000	0,00	0,30	0	0
2	0	0,6	0,150	0,13	0,30	0,01875	0,045
3	0,25	0,6	0,000	0,25	0,43	0	0
4	0,25	0,25	0,113	0,48	0,13	0,05344	0,0141
5	0,7	0,25	0,000	0,70	0,13	0	0
6	0,7	0	0,000	0,35	0,00	0	0
		Total:	0,263			0,0722	0,0591

Where:

edge_i = edge between corners: i and i+1

Area_c_i = Area between edge_i and axes: $x=x_i$, $x=x_{i+1}$ and $y=0$

$$\text{Area}_c = 1/2 \cdot (y_{i+1} + y_i) \cdot (x_{i+1} - x_i)$$

$x_{01_i} = x_i + (x_{i+1} - x_i) \cdot (2/3 \cdot y_{i+1} + 1/3 \cdot y_i) / (y_i + y_{i+1})$ if $(y_i + y_{i+1}) \neq 0$ or

$$x_{01_i} = (x_{i+1} + x_i) / 2 \text{ if } (y_i + y_{i+1}) = 0$$

$y_{01_i} = y_i / 2 + 0.5 \cdot (x_{01_i} - x_i) \cdot (y_{i+1} - y_i) / (x_{i+1} - x_i)$ if $x_{i+1} - x_i \neq 0$ or

$$y_{01_i} = (y_{i+1} + y_i) / 2 \text{ if } x_{i+1} - x_i = 0$$

$$S_{cy_i} = \text{Area}_c \cdot x_{01_i}$$

$$S_{cx_i} = \text{Area}_c \cdot y_{01_i}$$

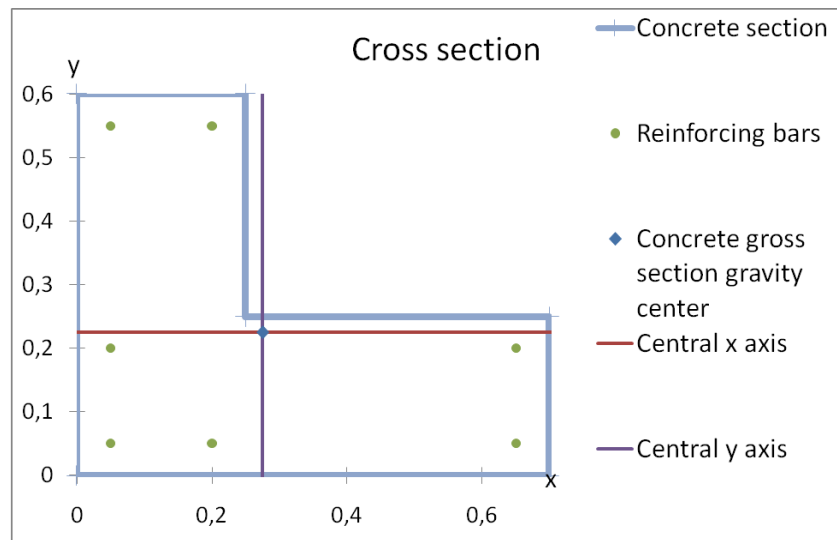
Concrete gross section area:

$$A_c = \sum(\text{Area}_c) = 0.263$$

Concrete gross section gravity center

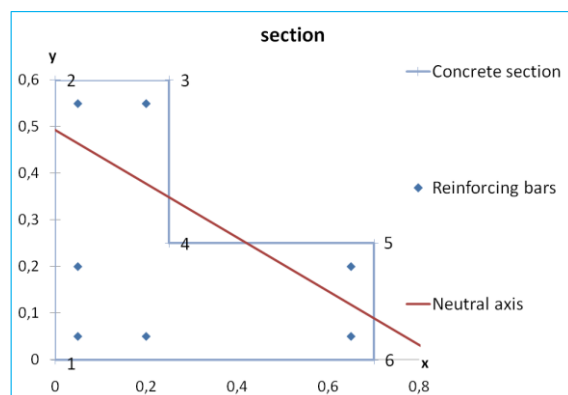
$$x_{Cc} = \sum(S_{cy_i}) / A_c = 0.275$$

$$y_{Cc} = \sum(S_{cx_i}) / A_c = 0.225$$



Equation of neutral axis in central axes coordinate system (x_{cc} ; y_{cc})

$$y_{cc} = -0.577350316 \cdot x_{cc} + 0.110000027 \text{ (e.g. obtained in iteration process)}$$



General equation of neutral axis in central axes coordinate system (x_{cc} ; y_{cc})

$$A \cdot x_{cc} + B \cdot y_{cc} + C = 0$$

$$A = -0.202$$

$$B = -0.350$$

$$C = 0.0385$$

Distance between corner of concrete section and neutral axis:

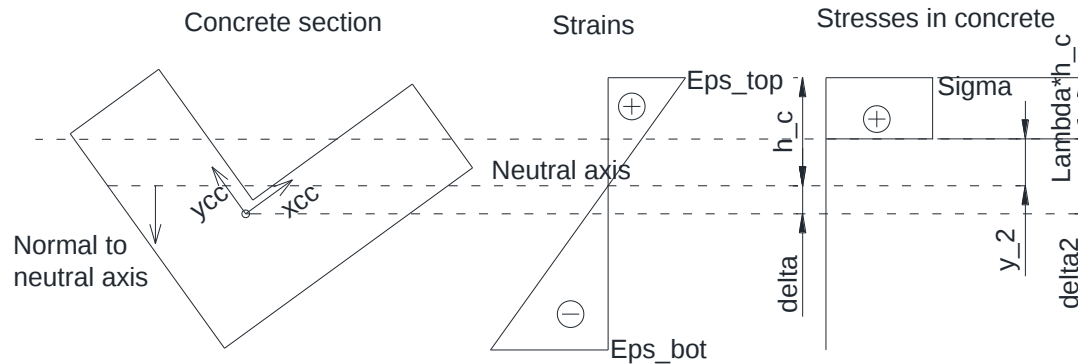
For tensioned corner:

$$\text{dist} = (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2 + B^2)^{0,5}$$

For compressed corner:

$$\text{dist} = - (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2 + B^2)^{0,5}$$

i	Corners of concrete section		Corners of concrete section in central axes coordinate system		Corner under compression/tension	Distance between corner and neutral axis dist
	x	y	x_{cc}	y_{cc}		
1	0	0	-0,275	-0,225	tension	0,428
2	0	0,6	-0,275	0,375	compression	-0,092
3	0,25	0,6	-0,025	0,375	compression	-0,217
4	0,25	0,25	-0,025	0,025	tension	0,086
5	0,7	0,25	0,425	0,025	compression	-0,139
6	0,7	0	0,425	-0,225	tension	0,078
Where:						
$x_{cc} = x - x_{Cc}$						
$y_{cc} = y - y_{Cc}$						



Height of compressed part of a section = maximum distance between compressed corner of the section and neutral axis:

$$h_c = |\min(\text{dist}_i)| = 0.217$$

Height of tensioned part of a section = maximum distance between tension corner of the section and neutral axis:

$$h_t = \max(\text{dist}_i) = 0.428$$

Verification of used equation of neutral axis:

$$\text{Eps}_{\text{bot}} = -h_t/h_c \cdot \text{Eps}_{\text{top}} = -0.006903226 \text{ (OK)}$$

and

$$\text{neutral axis angle} = \arctan(-0.577350316) + \pi/2 + \pi = 4.18879 = 240 \text{ deg} = \text{Angle (OK)}$$

- Results for reinforcement bars:

$$\text{Eps}_{u1} = f_{yd} / E_s$$

Distance between reinforcing bar and neutral axis:

For tensioned rebar:

$$d = (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2 + B^2)^{0.5}$$

For compressed rebar:

$$d = -(A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2 + B^2)^{0.5}$$

Stress in bar i:

If $|\text{eps}_i| \leq f_{yd} / E_s$ then

$$\text{sigma}_i = \text{eps}_i / \text{Eps}_{u1} \cdot f_{yd}$$

else

$$\text{sigma}_i = f_{yd} + (|\text{eps}_i| - \text{Eps}_{u1}) / (\text{Eps}_{u2} - \text{Eps}_{u1}) \cdot (k - 1) \cdot f_{yd}$$

i	x	y	ϕ	Positions in central axes coordinate system		Distance d from neutral axis	strain $\text{eps} = d/h_c \cdot \text{Eps}_{\text{top}}$	stresses $\text{sigma} / 10^6$	force in bar $N_s = \text{sigma} \cdot \pi \cdot \phi^2 / 4$	$M_{xs} = -N \cdot y_{cc}$	$M_{ys} = N \cdot x_{cc}$
				x_{cc}	y_{cc}						
1	0,05	0,05	0,02	-0,23	-0,18	0,359	-0,00580	-310	-97389	-17043	21913
2	0,05	0,55	0,02	-0,23	0,33	-0,074	0,00119	238	74685	-24273	-16804
3	0,2	0,55	0,02	-0,08	0,33	-0,149	0,00240	310	97389	-31652	-7304
4	0,2	0,05	0,02	-0,08	-0,18	0,284	-0,00459	-310	-97389	-17043	7304

5	0,65	0,05	0,02	0,38	-0,18	0,059	-0,00096	-191	-60114	-10520	-22543
6	0,65	0,2	0,02	0,38	-0,03	-0,071	0,00114	228	71535	1788	26826
7	0,05	0,2	0,02	-0,23	-0,03	0,229	-0,00370	-310	-97389	-2435	21913
Sum of forces in bars:								-108673	101177	-	31304

$$N_s = -108.67 \text{ e3}$$

$$M_{xs} = -101.18 \text{ e3}$$

$$M_{ys} = 31.30 \text{ e3}$$

Reinforcement bars lying in the zone of compressive stresses in concrete reduce area of compressed concrete (see fig. 3 on drawing in Case 2a). Reduction of forces carried by the concrete section is calculated as follows:

$$\text{if } \epsilon_s \leq \epsilon_{sc1} \quad N_{sc} = f_{cd} \cdot \epsilon_s / \epsilon_{sc1} \cdot \text{Area}$$

$$\text{if } \epsilon_s > \epsilon_{sc1} \quad N_{sc} = f_{cd} \cdot \text{Area}$$

i	x	y	ϕ	strain eps	Positions in central axes coordinate system		Area of concrete with compression stress cut from the section in place of bars	Force N_{sc} in concrete in place of bars	$M_{xsc} = -$ $N_{sc} \cdot y_{cc}$	$M_{ysc} =$ $N_{sc} \cdot x_{cc}$
					x_{cc}	y_{cc}				
1	0,05	0,05	0,02	-0,00580	-0,23	-0,18				
2	0,05	0,55	0,02	0,00119	-0,23	0,33	0,000314159	6283	-2042	-1414
3	0,2	0,55	0,02	0,00240	-0,08	0,33	0,000314159	6283	-2042	-471
4	0,2	0,05	0,02	-0,00459	-0,08	-0,18				
5	0,65	0,05	0,02	-0,00096	0,38	-0,18				
6	0,65	0,2	0,02	0,00114	0,38	-0,03	0,000314159	6283	157	2356
7	0,05	0,2	0,02	-0,00370	-0,23	-0,03				
Total:							0,000942478	18850	-3927	471

Reduction forces in concrete:

$$N_{sc} = 18.850 \text{ e3}$$

$$M_{xsc} = -3.927 \text{ e3}$$

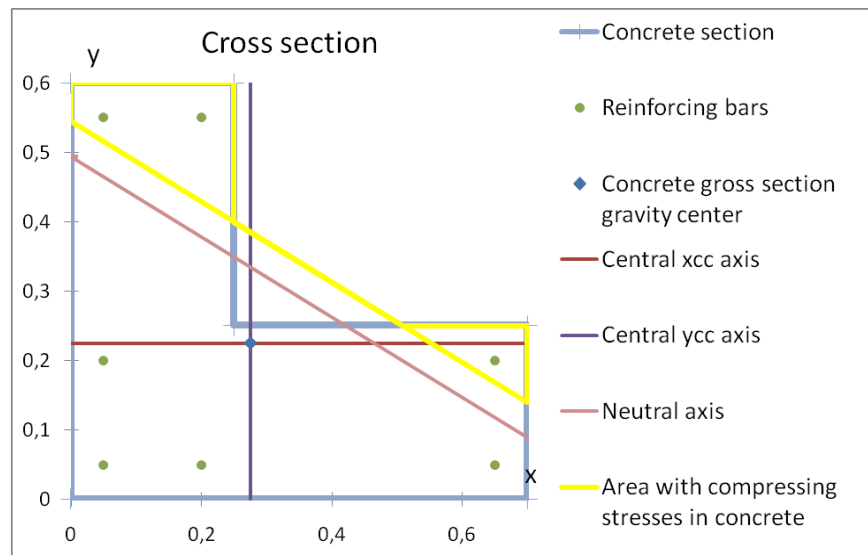
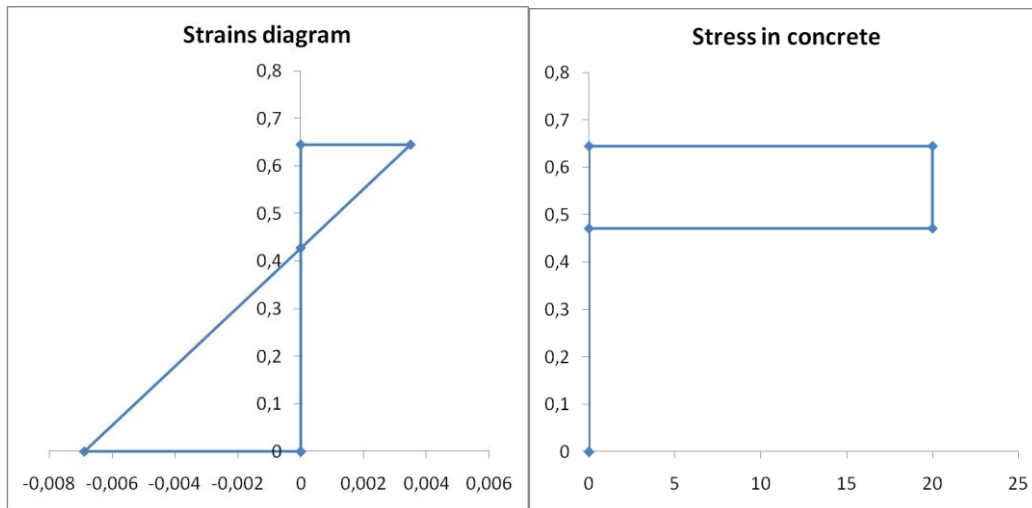
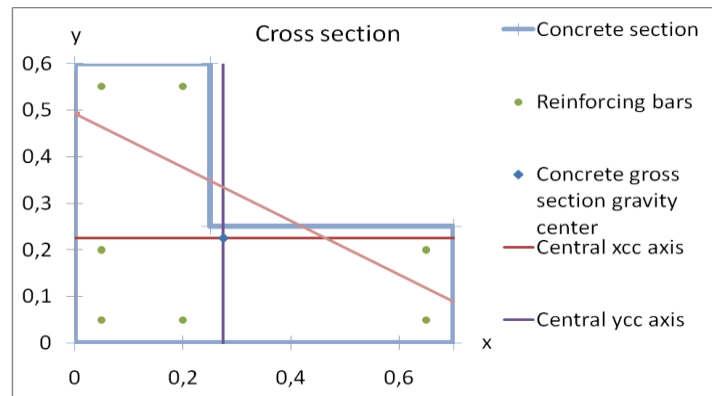
$$M_{ysc} = 0.471 \text{ e3}$$

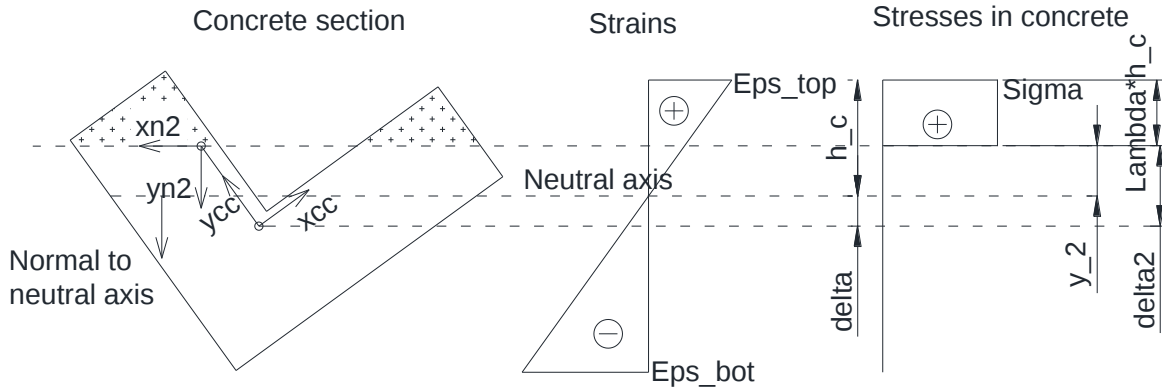
Center of gravity of stresses which would be in the concrete in place of rebars mentioned above (see also fig. 3 on drawing in case 2a) in central axes coordinate system:

$$x_{scg} = \sum(x_{cc_i} \cdot \text{Area_i}) / \sum(\text{Area_i}) = 0.0250$$

$$y_{scg} = \sum(y_{cc_i} \cdot \text{Area_i}) / \sum(\text{Area_i}) = 0.2083$$

- Results for concrete section:





$$\text{Sigma} = f_{cd} = 20.000 \text{ e6}$$

Distance between gravity center of contour and neutral axis:

$$\text{delta} = (A \cdot x_{cc} + B \cdot y_{cc} + C) / (A^2 + B^2)^{0.5}$$

for $x_{cc} = y_{cc} = 0$ and coefficients A, B, C as defined in paragraph: Position of neutral axis, above.

$$\text{delta} = 0.09526$$

Distance between the gravity center of contour of the section and area with compressive stresses in concrete:

$$\text{delta2} = \text{delta} + (1 - \text{Lambda}) \cdot h_c = 0.138862$$

To calculate a sum of stresses over the contour with compressive stresses in concrete we define a new coordinate system x_{n2} , y_{n2} (see picture above), where x_{n2} is an axis parallel to neutral axis, in distance delta2 from the gravity center of contour of the section, and it crosses axis y_{n2} in point (x_{n2c}, y_{n2c}) given in central axis coordinate system.

Equation of neutral axis in central axes coordinate system:

$$y_{cc} = -0.577350316 \cdot x_{cc} + 0.110000027$$

Equation of x_{n2} axis in central axes coordinate system

$$y_{nc} = -0.577350316 \cdot x_{nc} + \text{delta2} / \cos(\text{Angle}) = -0.577350316 \cdot x_{nc} + 0.16011327$$

For $x_{n2c} = 0$ and $y_{n2c} = 0.16011327$

Now we find coordinates of contour under stress solid in (x_{cc}, y_{cc}) coordinates system.

i	Contour		Contour in central axes coordinate system		Equation of line containing edge_i, in central axes coordinate system	Intersection point of line containing edge_i and x_{n2} axis		Is intersection on the contour?
	x	y	x_{cc}	y_{cc}		x_{cc}	y_{cc}	
1	0	0	-0,275	-0,225	$x_{cc} = -0,275$	-0,275	0,3189	Yes
2	0	0,6	-0,275	0,375	$y_{cc} = 0,375$	-0,3722	0,375	No
3	0,25	0,6	-0,025	0,375	$x_{cc} = -0,025$	-0,025	0,1745	Yes
4	0,25	0,25	-0,025	0,025	$y_{cc} = 0,025$	0,2340	0,025	Yes
5	0,7	0,25	0,425	0,025	$x_{cc} = 0,425$	0,425	-0,0853	Yes
6	0,7	0	0,425	-0,225	$y_{cc} = -0,225$	0,6670	-0,225	No
7	0	0	-0,275	-0,225	$x_{cc} = -0,275$	-0,275	0,3189	Yes

Where:

$$x_{cc} = x - x_{cc}$$

$$y_{cc} = y - y_{cc}$$

edge_i = edge between corners: i and i+1

Now we calculate area and gravity center of contour under compressive stresses in concrete in (x_{cc}, y_{cc}) coordinates system:

i	Contour with compressive stresses in concrete		Contour with compressive stresses in concrete		Area_c	Gravity center of contour corresponding to Area_c		Static moments	
	x_{cc}	y_{cc}	x	y		x_{01}	y_{01}	S_{cy}	S_{cx}
1	-0,275	0,375	0,000	0,600	0,1500	0,125	0,3	0,019	0,045
2	-0,025	0,375	0,250	0,600	0,0000	0,25	0,6	0	0
3	-0,025	0,175	0,250	0,400	0,0841	0,370	0,165	0,031	0,014
4	0,234	0,025	0,509	0,250	0,0477	0,605	0,125	0,029	0,006
5	0,425	0,025	0,700	0,250	0,0000	0,7	0,25	0	0
6	0,425	-0,085	0,700	0,140	-0,2393	0,281	0,191	-0,067	-0,046
7	-0,275	0,319	0,000	0,544	0,0000	0	0,544	0	0
8	-0,275	0,375	0,000	0,000	0,0000	0,125	0,3	0	0
				Total	0,0426			0,0115	0,0192

Where:

edge_i = edge between corners: i and i+1

Area_c_i = Area between edge_i and axes: $x=x_i$, $x=x_{(i+1)}$ and $y=0$

$$\text{Area}_{c_i} = 1/2 \cdot (y_{(i+1)} + y_{(i)}) \cdot (x_{(i+1)} - x_{(i)})$$

$x_{01_i} = x_{(i)} + (x_{(i+1)} - x_{(i)}) \cdot (2/3 \cdot y_{(i+1)} + 1/3 \cdot y_{(i)}) / (y_{(i)} + y_{(i+1)})$ if $(y_{(i)} + y_{(i+1)}) \neq 0$ or

$$x_{01_i} = (x_{(i+1)} + x_{(i)}) / 2 \text{ if } (y_{(i)} + y_{(i+1)}) = 0$$

$y_{01_i} = y_{(i)} / 2 + 0.5 \cdot (x_{01_i} - x_{(i)}) \cdot (y_{(i+1)} - y_{(i)}) / (x_{(i+1)} - x_{(i)})$ if $x_{(i+1)} - x_{(i)} \neq 0$ or

$$y_{01_i} = (y_{(i+1)} + y_{(i)}) / 2 \text{ if } x_{(i+1)} - x_{(i)} = 0$$

$$S_{cy_i} = \text{Area}_{c_i} \cdot x_{01_i} - y_{cc}$$

$$S_{cx_i} = \text{Area}_{c_i} \cdot y_{01_i} - x_{cc}$$

Compressed concrete area:

$$A_c = \text{sum}(\text{Area}_{c_i}) = 0.0426$$

Gravity center of compressed concrete contour (gross) in central axes coordinate system

$$x_c = \text{sum}(S_{cy_i}) / A_c - x_{cc} = -0.0060$$

$$y_c = \text{sum}(S_{cx_i}) / A_c - y_{cc} = -0.2260$$

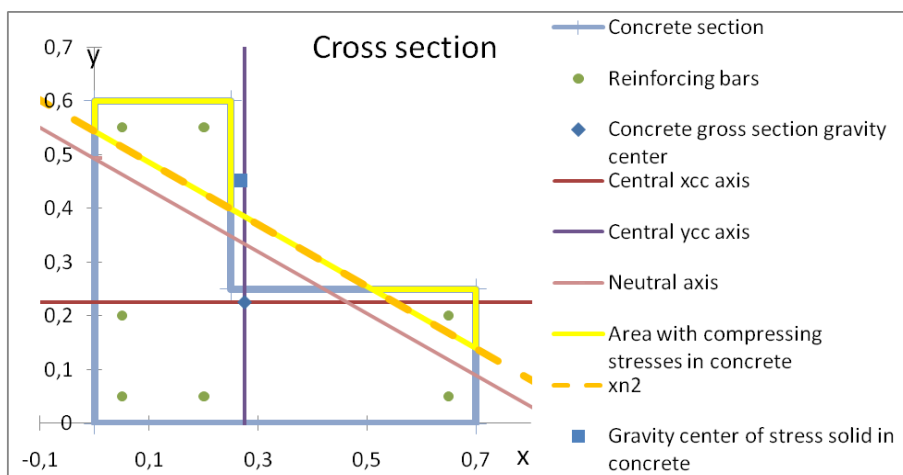
Axial force in compressed part of concrete gross section

$$N_{c1} = A_c \cdot \sigma = 0.851993 \text{ e6}$$

Center of gravity of stresses in concrete in central axes coordinate system:

$$x_{cg} = (x_c \cdot N_c - x_{scg} \cdot N_{sc}) / (N_c - N_{sc}) = -0.006674$$

$$y_{cg} = (y_c \cdot N_c - y_{scg} \cdot N_{sc}) / (N_c - N_{sc}) = 0.226430$$



Internal forces in concrete:

$$N_c = N_{c'} - N_{sc} = 833.14 \text{ e3}$$

$$M_{xc} = -N_c \cdot y_{cg} = -188.65 \text{ e3}$$

$$M_{yc} = N_c \cdot x_{cg} = -5.56 \text{ e3}$$

- Results for reduced section:

$$N_f = N_c + N_s = 724.47 \text{ e3}$$

$$M_{xf} = M_{xc} + M_{xs} = -289.83 \text{ e3}$$

$$M_{yf} = M_{yc} + M_{ys} = 25.74 \text{ e3}$$

Small differences between results from Component and from manual calculations are below 1.5% and are due to precision of iterations.

16.4 Case 15b

Comparison of capacity forces for various models of concrete (obtained in Component / in manual calculations) for example as in Case 15a:

stress-strain model of concrete behavior:	rectangular	linear	bilinear	parabolic-rectangular	power-rectangular
parameters of concrete	as in Case 15a $f_{cd} = 20\text{e6}$	$E_{ps_ult} = 0,0035$ $f_{cd} = 20\text{e6}/0.8 = 25\text{e6}$	$E_{ps_ult} = 0,0035$ $E_{ps_c1} = 0,0020$ $f_{cd} = 20\text{e6}/0.8 = 25\text{e6}$	$E_{ps_ult} = 0,0035$ $E_{ps_c1} = 0,0020$ $f_{cd} = 20\text{e6}/0.8 = 25\text{e6}$	$E_{ps_ult} = 0,0035$ $E_{ps_c1} = 0,0020$ $n = 1.5$ $f_{cd} = 20\text{e6}/0.8 = 25\text{e6}$
$N_f / \text{e3}$	725.91 / 724.47	658.70 / 657.37	737.18 / 736.31	760.14 / 759.21	751.24 / 750.30
$M_{xf} / \text{e3}$	-290.09 / -289.83	-263.20 / -262.98	-294.66 / -294.56	-303.83 / -303.72	-300.26 / -300.16
$M_{yf} / \text{e3}$	25.68 / 25.74	23.29 / 23.35	26.24 / 26.17	27.06 / 26.98	26.74 / 26.66

17 Case 16: Calculation of capacity state in symmetric section for fixed axial force

17.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, loads applied to the section (N) and give state of strain and stress at the failure for fixed axial force N and M_x or $M_y = 0$.

17.2 Summary:

This sample gives as results the internal forces for:

- A symmetric concrete section with reinforcement bars
- A rectangular model of concrete stress-strain relation
- A horizontal branch model of steel
- A bending in x direction (M_x) (Case 16a)
- A bending in y direction (M_y) (Case 16b)
- A fixed axial force (N).

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel
- The loads applied to the section

Based on input data, Component gives as output results:

- Results for reinforcement bars:
 - Axial force (N_s)
 - Moment with respect to x_{cc} axis (M_{xs})
 - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
 - Area of contour under compressive stress (A_c)
 - Center of gravity of stresses in the concrete (x_{cg} , y_{cg})
 - Axial force (N_c)
 - Moment with respect to x_{cc} axis (M_{xc})
 - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
 - Axial force at the failure related to acting forces ($N_f = \alpha \cdot N$)
 - Moment at the failure related to acting forces, with respect to x_{cc} axis ($M_{xf} = \alpha \cdot M_x$)
 - Moment at the failure related to acting forces, with respect to y_{cc} axis ($M_{yf} = \alpha \cdot M_y$)

Where alpha is a coefficient called "safety factor".
- Neutral axis position:
 - Distance between the axis and a gravity center of concrete gross section (dist)
 - Angle between horizontal axis and normal to neutral axis directed to the most tensioned fiber (Angle)
- Strains
 - Extreme strains in concrete (Eps_top, Eps_bot)
 - Extreme strains in steel (Eps_stop, Eps_sbot)

Where x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

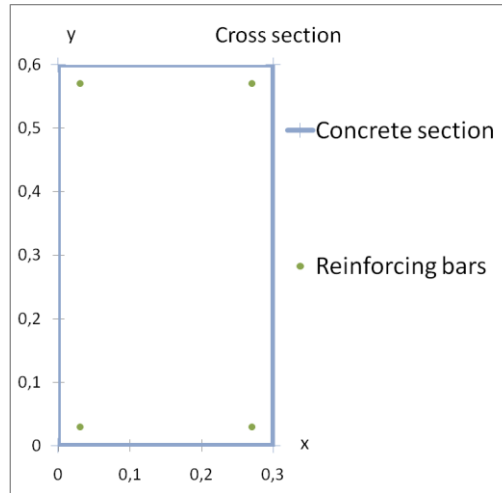
17.3 Case 16a

17.3.1 Data:

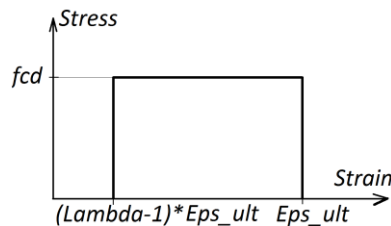
- Geometry:
 - Concrete section:

$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.6$
$x = 0.3$	$y = 0.6$
$x = 0.3$	$y = 0.0$
 - Reinforcing bars:

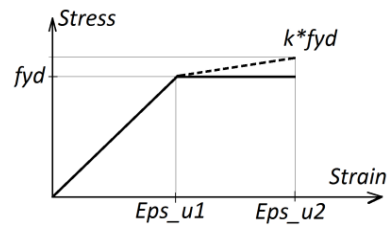
$x = 0.03$	$y = 0.03$	$\phi = 0.04$
$x = 0.03$	$y = 0.57$	$\phi = 0.04$
$x = 0.27$	$y = 0.57$	$\phi = 0.04$
$x = 0.27$	$y = 0.03$	$\phi = 0.04$



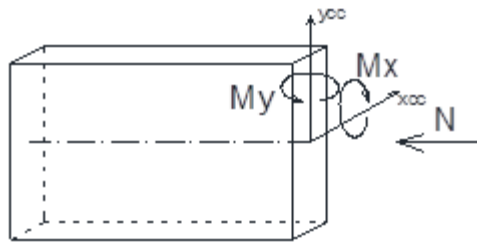
- Concrete parameters:
 - Design strength $f_{cd} = 21.4 \text{ e6} \cdot 0.8 = 17.12 \text{ e6}$
 - Effective height reduction factor $\Lambda = 0.8$
 - Modulus of elasticity $E_c = 32 \text{ e9}$
 - Strain-stress model: rectangular
 - Strain ultimate limit $Eps_ult = 0.0035$



- Steel parameters:
 - Design strength $f_{yd} = 310 \text{ e6}$
 - Hardening factor $k = 1.0$
 - Modulus of elasticity $E_s = 200 \text{ e9}$
 - Strain ultimate limit $Eps_u2 = 0.025$



- Forces:
 - $N = 678.00e3$
 - M_x
 - M_y



17.3.2 Search for:

- Results for reduced section:
 - Forces: $N_f = 678.00e3$, M_{xf} , M_{yf}
 - Extreme strains in concrete (Eps_top, Eps_bot)
 - Extreme strains in steel (Eps_top, Eps_bot)

17.3.3 Results from Component

- Results for reduced section:
 - $N_f = 677.98 e3$
 - $M_{xf} = -574.80 e3$
 - $M_{yf} = 0.00 e3$
 - Eps_top = 0.003500
 - Eps_bot = -0.008467
 - Eps_stop = 0.002902
 - Eps_sbot = -0.007869

Results from Component are consistent with a graph from book *“Podstawy projektowania konstrukcji żelbetowych i sprężonych według Eurokodu 2”*, Sekcja Konstrukcji Betonowych KILiW PAN, Wrocław 2006, page 360., ISBN 83-7125-136-X.

These results are also consistent with results from a program *spColumn v4.20*, STRUCTUREPOINT, LLC with precision over 99%.

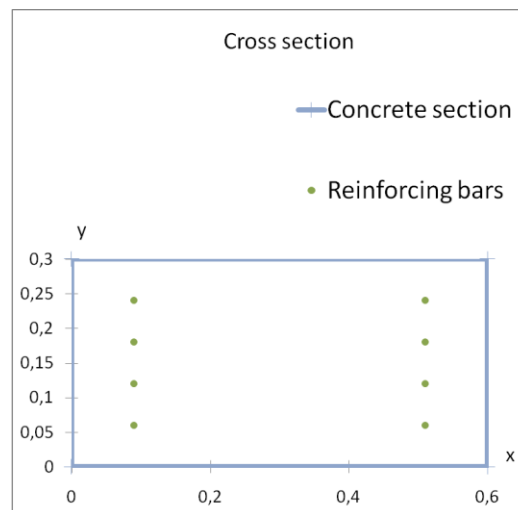
17.4 Case 16b

17.4.1 Data:

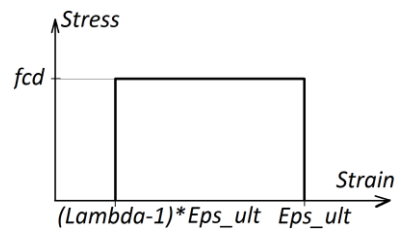
- Geometry:
 - Concrete section:

x = 0.0	y = 0.0
x = 0.0	y = 0.3
x = 0.6	y = 0.3
x = 0.6	y = 0.0
 - Reinforcing bars:

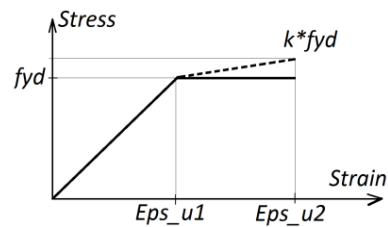
x = 0.09	y = 0.06	$\phi = 0.036$
x = 0.09	y = 0.12	$\phi = 0.036$
x = 0.09	y = 0.18	$\phi = 0.036$
x = 0.09	y = 0.24	$\phi = 0.036$
x = 0.51	y = 0.06	$\phi = 0.036$
x = 0.51	y = 0.12	$\phi = 0.036$
x = 0.51	y = 0.18	$\phi = 0.036$
x = 0.51	y = 0.24	$\phi = 0.036$



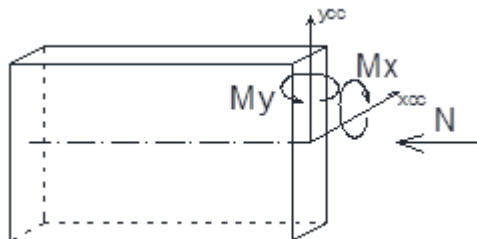
- Concrete parameters:
 - Design strength $f_{cd} = 21.4 \text{ e6} \cdot 0.8 = 17.12 \text{ e6}$
 - Effective height reduction factor $\Lambda = 0.8$
 - Modulus of elasticity $E_c = 32 \text{ e9}$
 - Strain-stress model: rectangular
 - Strain ultimate limit $Eps_{ult} = 0.0035$



- Steel parameters:
 - Design strength $f_{yd} = 420\text{e}6$
 - Hardening factor $k = 1.0$
 - Modulus of elasticity $E_s = 200\text{e}9$
 - Strain ultimate limit $Eps_u2 = 0.025$



- Forces:
 - $N = 1700.00\text{e}3$
 - M_x
 - M_y



17.4.2 Search for:

- Results for reduced section:
 - Forces: $N_f = 1700.00\text{e}3$, M_{xf} , M_{yf}
 - Extreme strains in concrete (Eps_top , Eps_bot)
 - Extreme strains in steel (Eps_top , Eps_bot)

17.4.3 Results from Component

- Results for reduced section:
 - $N_f = 1700.07\text{ e}3$
 - $M_{xf} = 0.00\text{ e}3$
 - $M_{yf} = 859.56\text{ e}3$
 - $Eps_top = 0.003500$
 - $Eps_bot = -0.002581$

Eps_stop = 0.002588
Eps_sbot = -0.001669

Results from Component are consistent with a graph from book *“Podstawy projektowania konstrukcji żelbetowych i sprężonych według Eurokodu 2”*, Sekcja Konstrukcji Betonowych KILiW PAN, Wrocław 2006, page 380., ISBN 83-7125-136-X. Differences between results are about 1%.

These results are also consistent with results from a program *spColumn v4.20*, STRUCTUREPOINT, LLC with precision over 99%.

18 Case 17: Calculation of capacity state in symmetric section for fixed moment

18.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, loads applied to the section (M_x or M_y) and give state of strain and stress at the failure for fixed moment (M_x or M_y) and another moment= 0.

18.2 Summary:

This sample gives as results the internal forces for:

- A symmetric concrete section with reinforcement bars
- A rectangular model of concrete stress-strain relation
- A horizontal branch model of steel
- A bending in x direction (M_x) (Case 17a) with fixed moment.
- A bending in y direction (M_y) (Case 17b) with fixed moment.

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel
- The loads applied to the section

Based on input data, Component gives as output results:

- Results for reinforcement bars:
 - Axial force (N_s)
 - Moment with respect to x_{cc} axis (M_{xs})
 - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
 - Area of contour under compressive stress (A_c)
 - Center of gravity of stresses in the concrete (x_{cg} , y_{cg})
 - Axial force (N_c)
 - Moment with respect to x_{cc} axis (M_{xc})
 - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
 - Axial force at the failure related to acting forces ($N_f = \alpha \cdot N$)
 - Moment at the failure related to acting forces, with respect to x_{cc} axis ($M_{xf} = \alpha \cdot M_x$)
 - Moment at the failure related to acting forces, with respect to y_{cc} axis ($M_{yf} = \alpha \cdot M_y$)

Where alpha is a coefficient called "safety factor".
- Neutral axis position:
 - Distance between the axis and a gravity center of concrete gross section (dist)
 - Angle between horizontal axis and normal to neutral axis directed to the most tensioned fiber (Angle)
- Strains
 - Extreme strains in concrete (Eps_top, Eps_bot)
 - Extreme strains in steel (Eps_stop, Eps_sbot)

Where x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

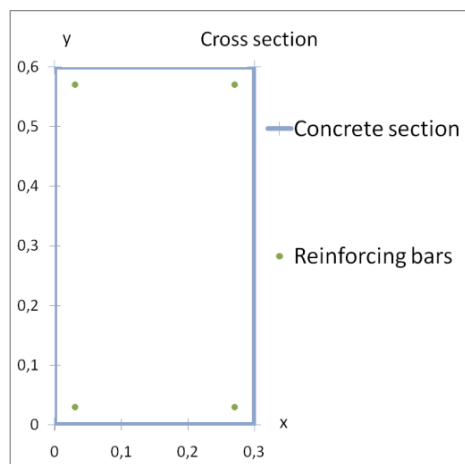
18.3 Case 17a

18.3.1 Data:

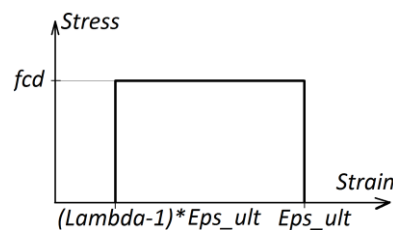
- Geometry:
 - Concrete section:

$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.6$
$x = 0.3$	$y = 0.6$
$x = 0.3$	$y = 0.0$
 - Reinforcing bars:

$x = 0.03$	$y = 0.03$	$\phi = 0.04$
$x = 0.03$	$y = 0.57$	$\phi = 0.04$
$x = 0.27$	$y = 0.57$	$\phi = 0.04$
$x = 0.27$	$y = 0.03$	$\phi = 0.04$

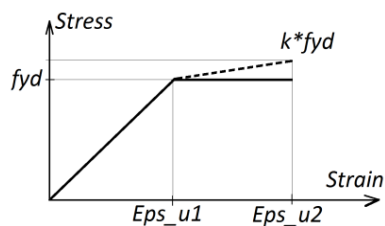


- Concrete parameters:
 - Design strength $f_{cd} = 21.4 \text{ e6} \cdot 0.8 = 17.12 \text{ e6}$
 - Effective height reduction factor $\Lambda = 0.8$
 - Modulus of elasticity $E_c = 32 \text{ e9}$
 - Strain-stress model: rectangular
 - Strain ultimate limit $Eps_ult = 0.0035$

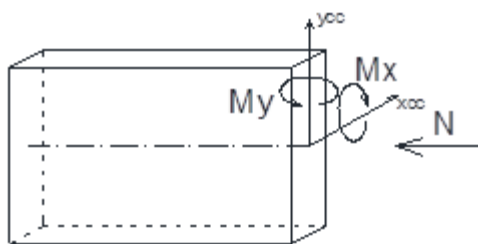


- Steel parameters:
 - Design strength $f_{yd} = 310 \text{ e6}$
 - Hardening factor $k = 1.0$

- Modulus of elasticity $E_s = 200e9$
- Strain ultimate limit $Eps_u2 = 0.025$



- Forces:
 - N
 - $M_x = 184.90e3$
 - M_y



18.3.2 Search for:

- Results for reduced section:
 - Forces: N_f , $M_{xf} = 184.90\ e3$, M_{yf}
 - Extreme strains in concrete (Eps_top , Eps_bot)
 - Extreme strains in steel (Eps_top , Eps_bot)

18.3.3 Results from Component

- Results for reduced section:
 - $N_f = 3876.03\ e3$
 - $M_{xf} = 184.90\ e3$
 - $M_{yf} = 0.00\ e3$
 - $Eps_top = 0.003500$
 - $Eps_bot = 0.000486$
 - $Eps_stop = 0.003349$
 - $Eps_sbot = 0.000637$

Results from Component are consistent with a graph from book *“Podstawy projektowania konstrukcji żelbetowych i sprężonych według Eurokodu 2”*, Sekcja Konstrukcji Betonowych KILiW PAN, Wrocław 2006, page 361., ISBN 83-7125-136-X. Differences between results are smaller than 1%. These results are also consistent with results from a program *spColumn v4.20*, STRUCTUREPOINT, LLC with precision over 99%.

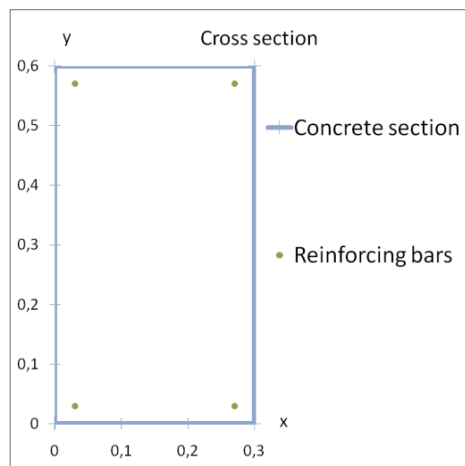
18.4 Case 17b

18.4.1 Data:

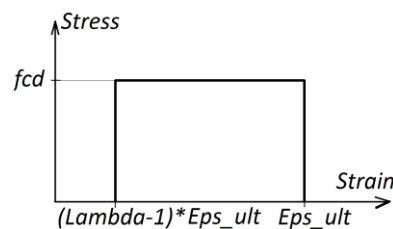
- Geometry:
 - Concrete section:

$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.6$
$x = 0.3$	$y = 0.6$
$x = 0.3$	$y = 0.0$
 - Reinforcing bars:

$x = 0.03$	$y = 0.03$	$\phi = 0.032$
$x = 0.03$	$y = 0.57$	$\phi = 0.032$
$x = 0.27$	$y = 0.57$	$\phi = 0.032$
$x = 0.27$	$y = 0.03$	$\phi = 0.032$

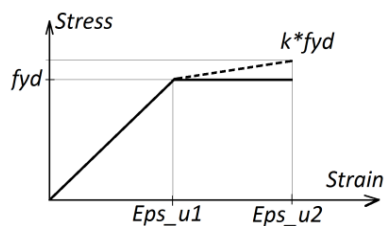


- Concrete parameters:
 - Design strength $f_{cd} = 21.4 \text{ e6} \cdot 0.8 = 17.12 \text{ e6}$
 - Effective height reduction factor $\Lambda = 0.8$
 - Modulus of elasticity $E_c = 32 \text{ e9}$
 - Strain-stress model: rectangular
 - Strain ultimate limit $Eps_ult = 0.0035$

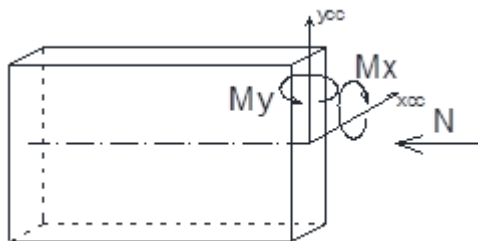


- Steel parameters:
 - Design strength $f_{yd} = 310 \text{ e6}$
 - Hardening factor $k = 1.0$
 - Modulus of elasticity $E_s = 200 \text{ e9}$

- Strain ultimate limit $Eps_u2 = 0.025$



- Forces:
 - N
 - Mx
 - $M_y = 138.67e3$



18.4.2 Search for:

- Results for reduced section:
 - Forces: N_f , M_{xf} , $M_{yf} = 138.67e3$
 - Extreme strains in concrete (Eps_top , Eps_bot)
 - Extreme strains in steel (Eps_top , Eps_bot)

18.4.3 Results from Component

- Results for reduced section:
 - $N_f = 2826.85 \text{ e3}$
 - $M_{xf} = 0.00 \text{ e3}$
 - $M_{yf} = 138.67 \text{ e3}$
 - $Eps_top = 0.003500$
 - $Eps_bot = -0.000204$
 - $Eps_stop = 0.003130$
 - $Eps_sbot = 0.000167$

Results from Component are consistent with a graph from book *“Podstawy projektowania konstrukcji żelbetowych i sprężonych według Eurokodu 2”*, Sekcja Konstrukcji Betonowych KILiW PAN, Wrocław 2006, page 363., ISBN 83-7125-136-X. Differences between results are smaller than 2%.

These results are also consistent with results from a program *spColumn v4.20*, STRUCTUREPOINT, LLC with precision over 99%.

19 Case 18: Calculation of capacity state in symmetric section for fixed negative (tensioning) axial force

19.1 Subject:

Read cross section geometry in specific point of element in the project, positions of reinforcing bars inside the section, parameters of concrete and steel, loads applied to the section (negative N) and give state of strain and stress at the failure for fixed axial force N and M_x or $M_y = 0$.

19.2 Summary:

This sample gives as results the internal forces for:

- A symmetric concrete section with reinforcement bars
- A rectangular model of concrete stress-strain relation
- A horizontal branch model of steel
- A bending in x direction (M_x) (Case 18a)
- A bending in y direction (M_y) (Case 18b)
- A fixed negative axial force (N).

Component needs as input data:

- The cross section geometry in specific point of element existing in the project
- The positions of reinforcing bars inside the section
- The parameters for the concrete and the steel
- The loads applied to the section

Based on input data, Component gives as output results:

- Results for reinforcement bars:
 - Axial force (N_s)
 - Moment with respect to x_{cc} axis (M_{xs})
 - Moment with respect to y_{cc} axis (M_{ys})
- Results for concrete section:
 - Area of contour under compressive stress (A_c)
 - Center of gravity of stresses in the concrete (x_{cg} , y_{cg})
 - Axial force (N_c)
 - Moment with respect to x_{cc} axis (M_{xc})
 - Moment with respect to y_{cc} axis (M_{yc})
- Results for reduced section:
 - Axial force at the failure related to acting forces ($N_f = \alpha \cdot N$)
 - Moment at the failure related to acting forces, with respect to x_{cc} axis ($M_{xf} = \alpha \cdot M_x$)
 - Moment at the failure related to acting forces, with respect to y_{cc} axis ($M_{yf} = \alpha \cdot M_y$)

Where alpha is a coefficient called "safety factor".
- Neutral axis position:
 - Distance between the axis and a gravity center of concrete gross section (dist)
 - Angle between horizontal axis and normal to neutral axis directed to the most tensioned fiber (Angle)
- Strains
 - Extreme strains in concrete (Eps_top, Eps_bot)
 - Extreme strains in steel (Eps_stop, Eps_sbot)

Where x_{cc} and y_{cc} are central axes of concrete gross section parallel to x and y axes.

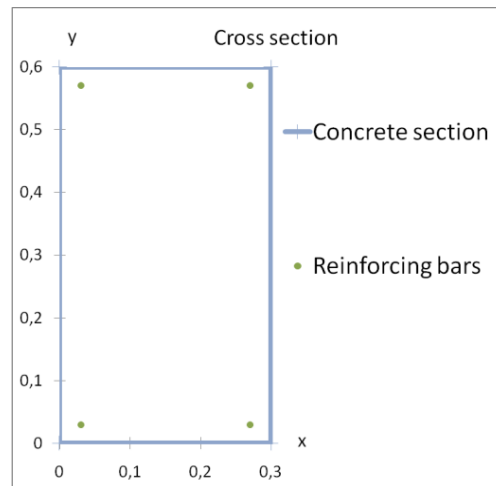
19.3 Case 18a

19.3.1 Data:

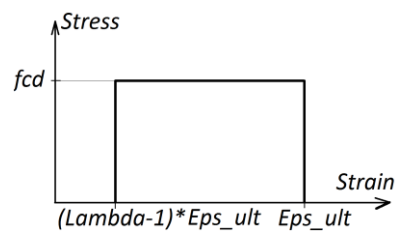
- Geometry:
 - Concrete section:

$x = 0.0$	$y = 0.0$
$x = 0.0$	$y = 0.6$
$x = 0.3$	$y = 0.6$
$x = 0.3$	$y = 0.0$
- Reinforcing bars:

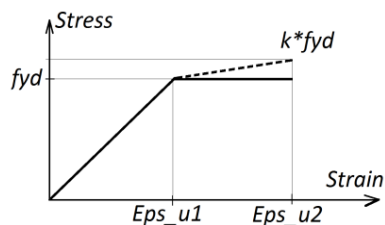
$x = 0.03$	$y = 0.03$	$\phi = 0.04$
$x = 0.03$	$y = 0.57$	$\phi = 0.04$
$x = 0.27$	$y = 0.57$	$\phi = 0.04$
$x = 0.27$	$y = 0.03$	$\phi = 0.04$



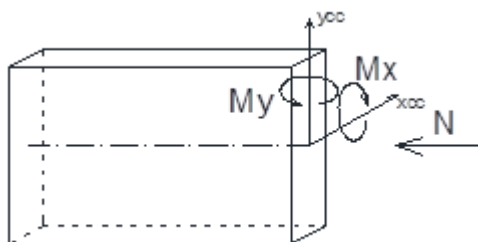
- Concrete parameters:
 - Design strength $f_{cd} = 21.4 \text{ e6} \cdot 0.8 = 17.12 \text{ e6}$
 - Effective height reduction factor $\Lambda = 0.8$
 - Modulus of elasticity $E_c = 32 \text{ e9}$
 - Strain-stress model: rectangular
 - Strain ultimate limit $Eps_ult = 0.0035$



- Steel parameters:
 - Design strength $f_{yd} = 310\text{e}6$
 - Hardening factor $k = 1.0$
 - Modulus of elasticity $E_s = 200\text{e}9$
 - Strain ultimate limit $Eps_u2 = 0.010$



- Forces:
 - $N = -493.06\text{ e}3$
 - M_x
 - M_y



19.3.2 Search for:

- Results for reduced section:
 - Forces: $N_f = -493.06\text{ e}3$, M_{xf} , M_{yf}
 - Extreme strains in concrete (Eps_top , Eps_bot)
 - Extreme strains in steel (Eps_top , Eps_bot)

19.3.3 Results from Component

- Results for reduced section:
 - $N_f = -493.05\text{ e}3$
 - $M_{xf} = -288.16\text{ e}3$
 - $M_{yf} = 0.00\text{ e}3$
 - $Eps_top = 0.001044$
 - $Eps_bot = -0.010581$
 - $Eps_stop = 0.000463$
 - $Eps_sbot = -0.010000$

Results from Component are consistent with a graph from book *“Podstawy projektowania konstrukcji żelbetonowych i sprężonych według Eurokodu 2”*, Sekcja Konstrukcji Betonowych KILiW PAN, Wrocław 2006, page 361., ISBN 83-7125-136-X. Differences between results are smaller than 3%.

These results are also consistent with results from a program *spColumn v4.20*, STRUCTUREPOINT, LLC with precision over 99%.

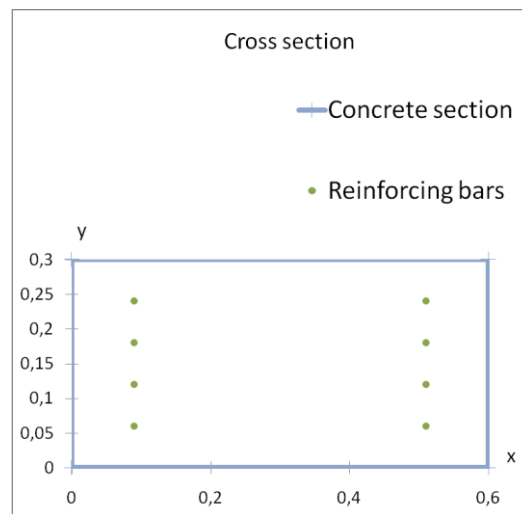
19.4 Case 18b

19.4.1 Data:

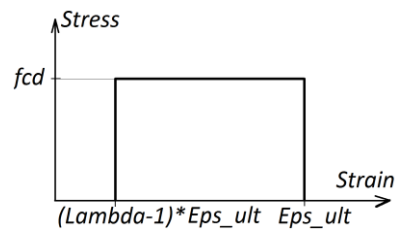
- Geometry:
 - Concrete section:

x = 0.0	y = 0.0
x = 0.0	y = 0.3
x = 0.6	y = 0.3
x = 0.6	y = 0.0
 - Reinforcing bars:

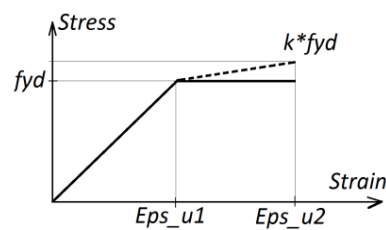
x = 0.09	y = 0.06	$\phi = 0.036$
x = 0.09	y = 0.12	$\phi = 0.036$
x = 0.09	y = 0.18	$\phi = 0.036$
x = 0.09	y = 0.24	$\phi = 0.036$
x = 0.51	y = 0.06	$\phi = 0.036$
x = 0.51	y = 0.12	$\phi = 0.036$
x = 0.51	y = 0.18	$\phi = 0.036$
x = 0.51	y = 0.24	$\phi = 0.036$



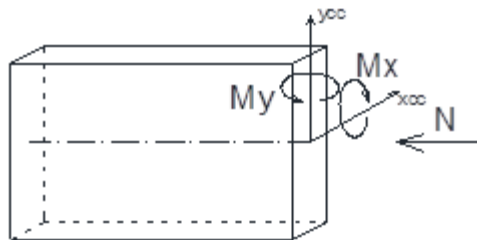
- Concrete parameters:
 - Design strength $f_{cd} = 21.4 \text{ e6} \cdot 0.8 = 17.12 \text{ e6}$
 - Effective height reduction factor $\Lambda = 0.8$
 - Modulus of elasticity $E_c = 32 \text{ e9}$
 - Strain-stress model: rectangular
 - Strain ultimate limit $Eps_{ult} = 0.0035$



- Steel parameters:
 - Design strength $f_{yd} = 420e6$
 - Hardening factor $k = 1.0$
 - Modulus of elasticity $E_s = 200e9$
 - Strain ultimate limit $Eps_u2 = 0.010$



- Forces:
 - $N = -862.85 \text{ e3}$
 - M_x
 - M_y



19.4.2 Search for:

- Results for reduced section:
 - Forces: $N_f = -862.85 \text{ e3}$, M_{xf} , M_{yf}
 - Extreme strains in concrete (Eps_top , Eps_bot)
 - Extreme strains in steel (Eps_top , Eps_bot)

19.4.3 Results from Component

- Results for reduced section:
 - $N_f = -862.80 \text{ e3}$
 - $M_{xf} = 0.00 \text{ e3}$
 - $M_{yf} = 554.30 \text{ e3}$
 - $Eps_top = 0.002908$
 - $Eps_bot = -0.012278$
 - $Eps_stop = 0.000630$

Eps_sbot = -0.010000

Results from Component are consistent with a graph from book *“Podstawy projektowania konstrukcji żelbetowych i sprężonych według Eurokodu 2”*, Sekcja Konstrukcji Betonowych KILiW PAN, Wrocław 2006, page 380., ISBN 83-7125-136-X. Differences between results are about 1%.

These results are also consistent with results from a program *spColumn v4.20*, STRUCTUREPOINT, LLC with precision over 99%.